



# basic education

Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

## **NATIONAL SENIOR CERTIFICATE**

**GRADE 12**

**MATHEMATICS P2**

**NOVEMBER 2010**

**MEMORANDUM**

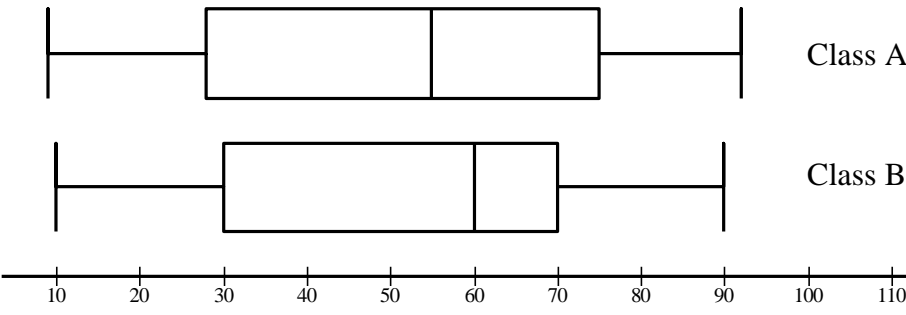
**MARKS: 150**

**This memorandum consists of 33 pages.**

**NOTE:**

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in ALL aspects of the marking memorandum.

**QUESTION 1**

<p>1.1</p>	<p>The Data Set</p> <p style="text-align: center;"> <math>Q_1</math>  <math>\downarrow</math>                  9 14 14 19 21 23 33 35 37 37 42 45 <math>Q_2</math>  <math>\downarrow</math>                  55 56 57 59 68 75  <math>Q_3</math>  <math>\downarrow</math>                  75 75 77 78 80 81 92             </p> <p>Min = 9 Max = 92 <math>Q_3</math> = Upper Quartile = 75 <math>Q_1</math> = Lower Quartile = 28</p> <p>(6<sup>th</sup> number is 23. 7<sup>th</sup> number is 33. The number in position 6,25 is <math>23 + \frac{1}{4}(33 - 23) = 25,5</math> ) (Accept <math>Q_1 = 25,5</math> )</p> <p><math>Q_2</math> = Median = 55</p> <p>Five number summary is (9 ; 28 ; 55 ; 75 ; 92) OR (9 ; 25,5 ; 55 ; 75 ; 92)</p>	<p>✓ min &amp; max ✓ upper quartile ✓ lower quartile ✓ median (4)</p> <p><b>Note:</b> Penalty 1 for not labelling or writing the numbers in ascending order</p>
<p>1.2</p>	 <p>Class A</p> <p>Class B</p> <p>10 20 30 40 50 60 70 80 90 100 110</p>	<p>✓ box ✓ whiskers (2)</p>

**Note:**

If the candidate combines the answer of 1.1 and 1.2 by drawing the correct box and whisker diagram in 1.2 and writes the numbers on the diagram (i.e. we mark question 1.1 and 1.2 in conjunction with one another), **max 5 / 6 marks for question 1.1 and 1.2**

If candidate writes numbers randomly in 1.1 and draws the box and whisker diagram correctly but without indicating the numbers on the diagram, then **max 5 / 6 marks for question 1.1 and 1.2**

If candidate just draws the box and whisker in 1.2 and does not indicate values on it or answers 1.1, the **max 2 / 6 marks**

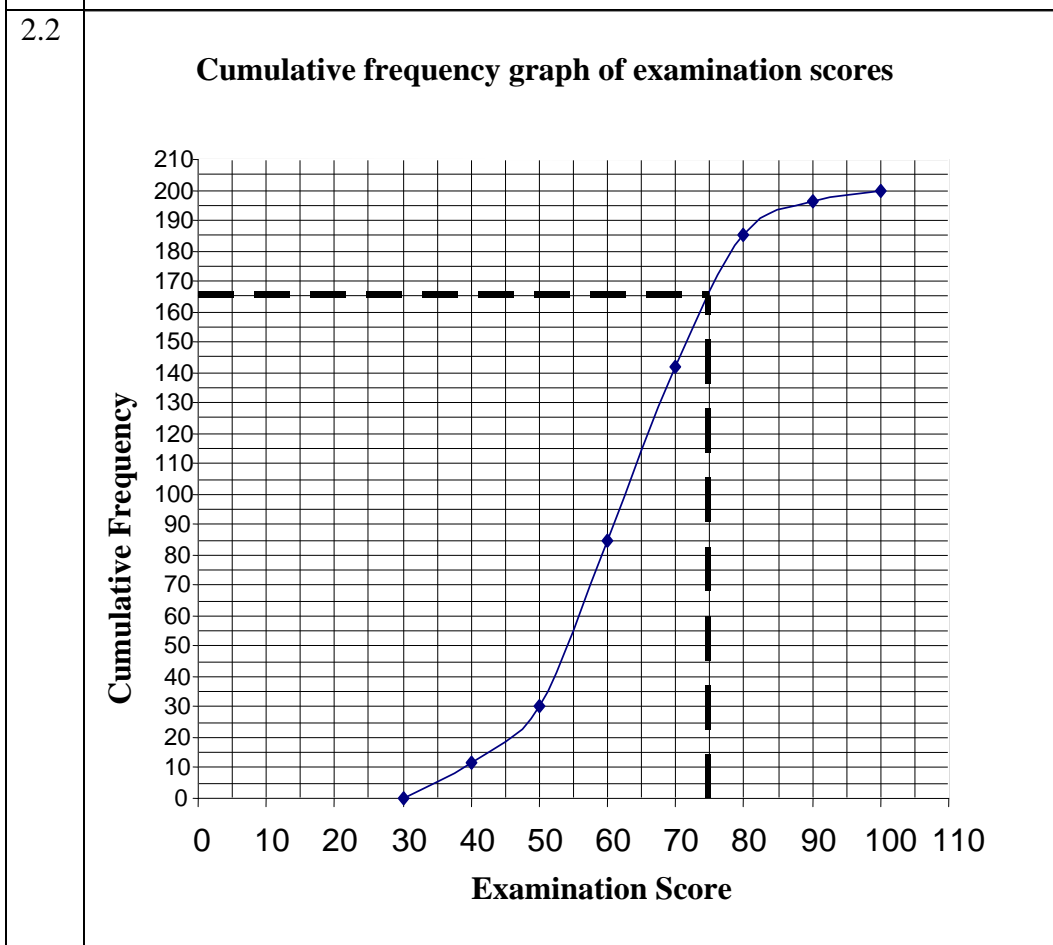
If the candidate draws two diagrams (one in the answer book and one on the diagram sheet), mark the one on the **DIAGRAM SHEET**.

<p>1.3</p>	<p>Class B Class B performed better because half of the learners got above 60% whilst half of Class A got more than 55%. Class B performed better because half of the learners got above 60% whilst half of Class A got less than 55%. Median of Class B &gt; Median of Class A</p> <p><b>OR</b> Class B Class B is skewed more to the left than Class A is.</p> <p><b>OR</b> Class A 25% of class scored 75% or more in Class A while 25% of the class scored 70% or more in Class B. Highest Mark in Class A &gt; Highest Mark in Class B.</p> <p><b>Note:</b> If candidate answers: Cannot determine the class that does better because we have insufficient information as we do not know where the marks are clustered. <b>Max 1 / 3</b></p> <p><b>Note:</b> If candidate just answers Class A or Class B and there are no reasons, then <b>0 / 3 marks</b></p>	<p>✓ Class B</p> <p>✓✓ median Class B &gt; Median Class A (3)</p> <p>✓ Class B ✓✓ Class B skewed more left than A (3)</p> <p>✓ Class A ✓ highest A &gt; highest B 25% of A above 75% and 25% of B above 70% (2)</p> <p>[9]</p>
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**QUESTION 2**

2.1	EXAMINATION SCORE (x)	FREQUENCY	CUMULATIVE FREQUENCY
	$30 \leq x < 40$	12	12
	$40 \leq x < 50$	18	30
	$50 \leq x < 60$	55	85
	$60 \leq x < 70$	57	142
	$70 \leq x < 80$	43	185
	$80 \leq x < 90$	11	196
	$90 \leq x < 100$	4	200

✓ first 3 values  
✓ last 4 values  
(2)



✓ shape (points must not be joined a straight line with a ruler)  
✓ grounding point (30 ; 0)  
✓ using the upper limit  
✓ using cumulative frequencies  
✓ if 4 or more points plotted correctly  
(5)

**Note:**

If learners plot the midpoint of the interval and the cumulative frequency **max 1 / 5 marks** for shape

If the candidates plot the lower limit and the cumulative frequency **max 1 / 5 marks**

**OR**

$$\frac{142 + 185}{2} = 163,5$$

$$200 - 163,5 = 36,5$$

**Note:**

Accept any one of 34, 35 or 36

✓ answer  
(1)  
**[8]**

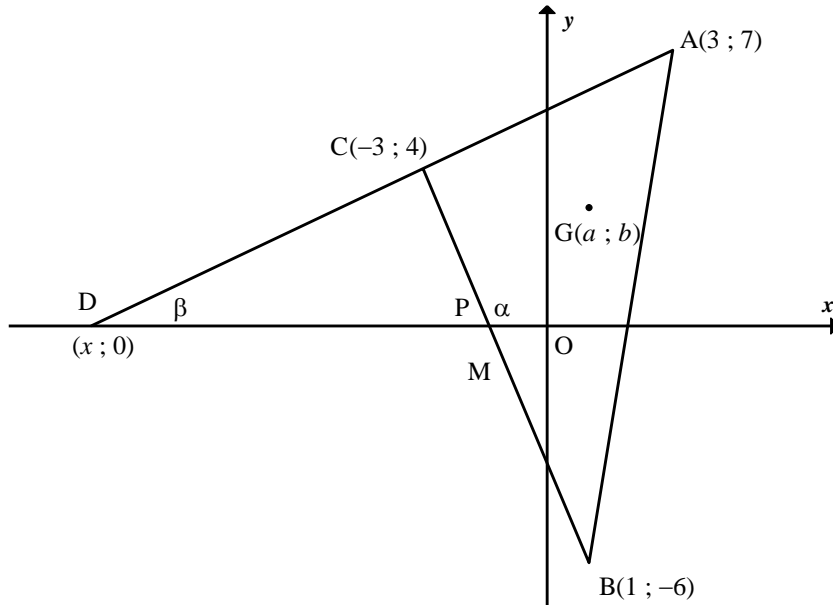
**QUESTION 3**

<p>3.1</p>	<p>Mean  <math display="block">= \frac{217 + 211 + 221 + 239 + 144 + 161 + 168 + 185 + 265 + 249 + 160 + 184}{12}</math> <math display="block">= \frac{2404}{12}</math> <math display="block">= 200,33</math></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> <p><b>Note:</b> Penalty 1 for incorrect rounding</p> </div>	<p>✓ <math>\frac{2404}{12}</math>                  ✓ answer (2)                  Answer only:                  Full marks</p>																																										
<p>3.2</p>	<p>By means of a calculator:  <math>\sigma = 37,37</math></p> <p><b>OR</b></p> <p><b>Pen and paper method:</b>                  mean (<math>\bar{x}</math>) = 200,33</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="width: 33%;">x</th> <th style="width: 33%;">x - <math>\bar{x}</math></th> <th style="width: 33%;">(x - <math>\bar{x}</math>)<sup>2</sup></th> </tr> </thead> <tbody> <tr><td>217</td><td>16,67</td><td>277,889</td></tr> <tr><td>211</td><td>10,67</td><td>113,848</td></tr> <tr><td>221</td><td>20,67</td><td>427,248</td></tr> <tr><td>239</td><td>38,67</td><td>1495,368</td></tr> <tr><td>144</td><td>-56,33</td><td>3173,0689</td></tr> <tr><td>161</td><td>-39,33</td><td>1546,848</td></tr> <tr><td>168</td><td>-32,33</td><td>1045,228</td></tr> <tr><td>185</td><td>-15,33</td><td>235,008</td></tr> <tr><td>265</td><td>64,67</td><td>4182,208</td></tr> <tr><td>249</td><td>48,67</td><td>2368,768</td></tr> <tr><td>160</td><td>-40,33</td><td>1626,508</td></tr> <tr><td>184</td><td>-16,33</td><td>266,668</td></tr> <tr> <td colspan="2">SUM</td> <td>16758,666</td> </tr> </tbody> </table> $\sigma = \sqrt{\frac{16758,6668}{12}} = 37,37$	x	x - $\bar{x}$	(x - $\bar{x}$ ) <sup>2</sup>	217	16,67	277,889	211	10,67	113,848	221	20,67	427,248	239	38,67	1495,368	144	-56,33	3173,0689	161	-39,33	1546,848	168	-32,33	1045,228	185	-15,33	235,008	265	64,67	4182,208	249	48,67	2368,768	160	-40,33	1626,508	184	-16,33	266,668	SUM		16758,666	<p>✓✓✓ answer (3)</p> <p>✓ total                  ✓ substitution                  ✓ answer (3)</p>
x	x - $\bar{x}$	(x - $\bar{x}$ ) <sup>2</sup>																																										
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<p>3.3</p>	<p>200,33 + 1(37,37)                  = 237,70 litres</p> <p>Accept any number between and including 237 and 238 litres.</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> <p><b>Note:</b>                      If candidate answers                      200,33 - 1(37,37) = 162,96 litres  <b>Max 1 / 2 marks</b></p> </div>	<p>✓ method                  ✓ answer (2)                  [7]</p>																																										

**QUESTION 4**

4.1	Fly High	✓ answer (1)
4.2	$\frac{5120}{1000} \times 7,9$ $= 40,45$ Yes <b>OR</b> $\frac{5120}{1000} \times 8$ $= 40,96$ <b>OR</b> $8 \times 5$ $= 40$	✓ calculation that leads to 40     (1)
4.3	Yes. The data points suggest a straight line fit with negative gradient but Fly-High will have to be an outlier.  <b>OR</b> Yes. Weak negative correlation. ( $r = -0,2128075984$ )  <b>Note:</b> If the candidate indicates “Best Air” and/or “Best Fly” and/or “Alpha” have high on time arrivals and low lost luggage <b>max 1 / 2 marks</b>	✓Yes ✓ negative gradient (2)  ✓Yes ✓ negative correlation (2)
4.4	Alpha, 70% on-time arrival and least luggage loss  <b>OR</b> Best Air, best on time arrival	✓ Name of company ✓correct justification (2) <b>[6]</b>

**QUESTION 5**



<p>5.1.1</p>	$m_{AD} = m_{AC}$ $= \frac{7-4}{3-(-3)}$ $= \frac{3}{6}$ $= \frac{1}{2}$ <p style="text-align: center;"><b>OR</b></p> $m_{AD} = m_{AC}$ $= \frac{4-7}{-3-(3)}$ $= \frac{-3}{-6}$ $= \frac{1}{2}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p><b>Note:</b> If candidate gives  <math display="block">m_{AD} = \frac{7}{3-x}</math>                      then <b>1/2 marks</b></p> </div>	<p>✓ substitution of A and C into correct formula</p> <p>✓ answer</p> <p style="text-align: right;">(2)</p>
<p>5.1.2</p>	$m_{BC} = \frac{-6-4}{1-(-3)}$ $= \frac{-10}{4}$ $= \frac{-5}{2}$ <p style="text-align: center;"><b>OR</b></p> $m_{BC} = \frac{4-(-6)}{-3-(1)}$ $= \frac{10}{-4}$ $= \frac{-5}{2}$	<p>✓ answer</p> <p style="text-align: right;">(1)</p>
<p>5.2</p>	$m_{AD} = \frac{1}{2} = \tan \hat{CDO}$ $\hat{CDO} = 26,56505\dots^\circ$ $m_{BC} = \frac{-5}{2} = \tan \alpha$ $\alpha = 111,814\ 095^\circ$ $\hat{DCB} = 111,8014095\dots^\circ - 26,56505\dots^\circ$ $= 85,236359^\circ$ $= 85,24^\circ$ $\approx 85,2^\circ$ <p style="text-align: left; margin-top: 20px;"><b>OR</b></p>	<p>✓ 26,57°</p> <p>✓ 111,80°</p> <p>✓ answer</p> <p style="text-align: right;">(3)</p>

$\tan \hat{CDO} = \frac{1}{2}$ $\hat{CDO} = 26,56505\dots^\circ$ $\tan(180^\circ - \alpha) = \frac{5}{2}$ $180^\circ - \alpha = 68,19859051\dots^\circ$ $\hat{DCB} = 180^\circ - (26,56505\dots^\circ + 68,19859051\dots^\circ)$ $= 85,236359^\circ$ $= 85,24^\circ$ <p><b>OR</b></p> $\hat{DCB} = \alpha - \hat{CDO}$ $\tan \hat{DCB} = \frac{m_{CB} - m_{CD}}{1 + m_{CB} \cdot m_{CD}}$ $= \frac{-\frac{5}{2} - \frac{1}{2}}{1 + \left(-\frac{5}{2}\right)\left(\frac{1}{2}\right)}$ $= 12$ $\hat{DCB} = 85,24^\circ$ <p><b>OR</b></p> $AC = \sqrt{45} \quad BC = \sqrt{116} \quad AB = \sqrt{173}$ $\cos \hat{ACB} = \frac{AC^2 + BC^2 - AB^2}{2AC \cdot BC}$ $= \frac{45 + 116 - 173}{2(\sqrt{45})(\sqrt{116})}$ $= -0,083045\dots$ $\hat{ACB} = 94,76\dots^\circ$ $\hat{DCB} = 180^\circ - 94,76\dots^\circ$ $= 85,24^\circ$ <p><b>OR</b></p> $D(-11; 0)$ $DC = \sqrt{80} \quad BC = \sqrt{116} \quad DB = \sqrt{180}$ $\cos \hat{DCB} = \frac{DC^2 + BC^2 - DB^2}{2DC \cdot BC}$ $= \frac{80 + 116 - 180}{2(\sqrt{80})(\sqrt{116})}$ $= 0,08304547985\dots$ $\hat{DCB} = 85,24^\circ$ <p><b>OR</b></p>	<p>✓ 26,57°</p> <p>✓ 68,2°</p> <p>✓ answer (3)</p> <p>✓</p> $\tan \hat{DCB} = \frac{m_{CB} - m_{CD}}{1 + m_{CB} \cdot m_{CD}}$ <p>✓ substitution</p> <p>✓ answer (3)</p> <p>✓ cosine rule</p> <p>✓ substitution into cosine rule</p> <p>✓ answer (3)</p> <p>✓ cosine formula</p> <p>✓ substitution into cosine rule</p> <p>✓ answer (3)</p>
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	<p>Equation AC: <math>2y = x + 11</math>  D(-11 ; 0)  C(-3 ; 4)  <math>DC^2 = (x_C - x_D)^2 + (y_C - y_D)^2</math>  <math>= (-3 + 11)^2 + (4 - 0)^2</math>  <math>= 80</math></p> <p>Equation BC: <math>2y = -5x - 7</math>  P(-<math>\frac{7}{5}</math>; 0)  <math>PC^2 = (-3 + \frac{7}{5})^2 + (4 - 0)^2</math>  <math>= \frac{464}{25}</math></p> <p><math>DP^2 = (-\frac{7}{5} + 11)^2</math>  <math>= \frac{2304}{25}</math></p> <p>In <math>\triangle DCP</math>: <math>DP^2 = DC^2 + CP^2 - 2DC \cdot CP \cdot \cos \hat{D}CP</math>  <math>\frac{2304}{25} = \frac{2000}{25} + \frac{464}{25} - 2\left(\frac{\sqrt{2000}}{5}\right)\left(\frac{\sqrt{464}}{5}\right) \cdot \cos \hat{D}CP</math>  <math>\hat{D}CP = 85,23635\dots</math>  <math>\hat{D}CP = 85,24^\circ</math></p>	<p>✓ cosine formula</p> <p>✓ substitution into cosine rule</p> <p>✓ answer</p> <p>(3)</p>
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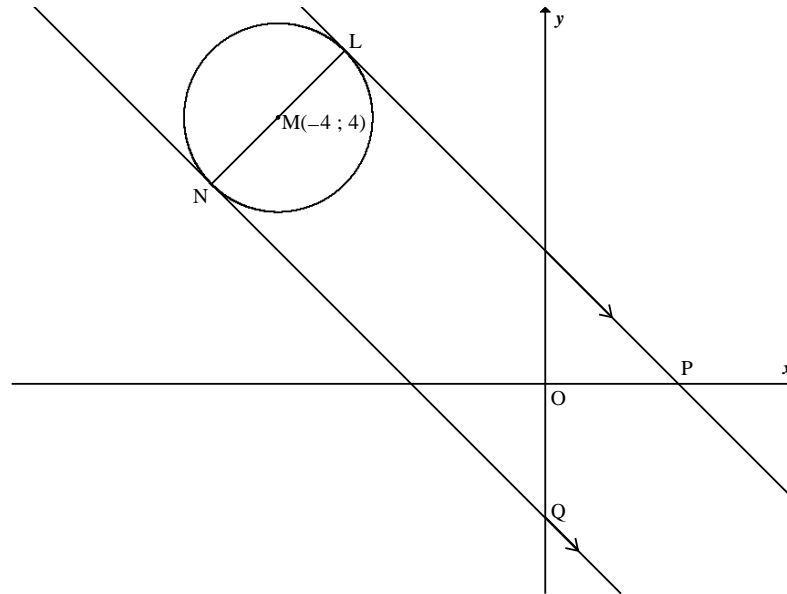
<p>5.3</p>	$y - 7 = \frac{1}{2}(x - 3)$ $y = \frac{1}{2}x + \frac{11}{2}$ $x - 2y + 11 = 0$ <p><b>OR</b></p> $y - 4 = \frac{1}{2}(x + 3)$ $y = \frac{1}{2}x + \frac{11}{2}$ $x - 2y + 11 = 0$ <p><b>OR</b></p> $y = \frac{1}{2}x + c$ $(7) = \frac{1}{2}(3) + c$ $c = \frac{11}{2}$ $y = \frac{1}{2}x + \frac{11}{2}$ $x - 2y + 11 = 0$	<p>✓ substitution of (3 ; 7) into <math>y - y_1 = m(x - x_1)</math></p> <p>✓ answer in any form (2)</p> <p>✓ substitution of (-3 ; 4) into <math>y - y_1 = m(x - x_1)</math></p> <p>✓ answer in any form (2)</p> <p>✓ substitution of (3 ; 7) into <math>y = mx + c</math></p> <p>✓ answer in any form (2)</p>
<p>5.4</p>	<p><math>M(x; y) = \left( \frac{-3+1}{2}; \frac{4-6}{2} \right)</math></p> <p><math>M(x; y) = (-1; -1)</math></p>	<p>✓ substitution</p> <p>✓ answer (2)</p>

**Note:**  
If candidate leaves answer as  $y - 7 = \frac{1}{2}(x - 3)$  or  $y - 4 = \frac{1}{2}(x + 3)$  :  
**max 1 / 3 marks**

<p>5.5</p> $m_{AM} = \frac{7 - (-1)}{3 - (-1)} = 2$ $y = 2x + c$ $-1 = 2(-1) + c$ $\therefore c = 1$ $y = 2x + 1$ <p><math>G(a ; b)</math> lies on the line</p> $\therefore b = 2a + 1$ <p><b>OR</b></p> $\frac{7 - b}{3 - a} = \frac{b + 1}{a + 1}$ $(7 - b)(a + 1) = (b + 1)(3 - a)$ $7a + 7 - ab - b = 3b - ab + 3 - a$ $8a - 4b = -4$ $2a - b = -1$ $b = 2a + 1$ <p><b>OR</b></p> <p>Using the point <math>(-1 ; -1)</math></p> $\frac{b + 1}{a + 1} = \frac{8}{4}$ $\frac{b + 1}{a + 1} = 2$ $b + 1 = 2a + 2$ $b = 2a + 1$ <p><b>OR</b></p> <p>Using the point <math>(3 ; 7)</math></p> $\frac{7 - b}{3 - a} = \frac{8}{4}$ $\frac{7 - b}{3 - a} = 2$ $7 - b = 6 - 2a$ $b = 2a + 1$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p><b>Note:</b> If the candidate does not conclude <math>b = 2a + 1</math> from <math>y = 2x + 1</math>: <b>max 3 / 4 marks</b></p> </div>	<p>✓ gradient = 2</p> <p>✓ substitution <math>(-1 ; -1)</math></p> <p>✓ <math>c = 1</math></p> <p>✓ conclusion (4)</p> <p>✓ <math>\frac{7 - b}{3 - a}</math></p> <p>✓ <math>\frac{b + 1}{a + 1}</math></p> <p>✓ equating</p> <p>✓ simplification leading to <math>2a - b = -1</math> (4)</p> <p>✓ substitution of <math>(-1 ; -1)</math> into gradient</p> <p>✓ gradient = 2</p> <p>✓ equating</p> <p>✓ simplification leading to <math>b + 1 = 2a + 2</math> (4)</p> <p>✓ substitution of <math>(3 ; 7)</math> into gradient</p> <p>✓ gradient = 2</p> <p>✓ equating</p> <p>✓ simplification leading to <math>7 - b = 6 - 2a</math> (4)</p>
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<p>5.6</p> <p><math>GC = \sqrt{17}</math></p> <p><math>GC^2 = 17</math></p> <p><math>(a + 3)^2 + (b - 4)^2 = 17</math></p> <p><math>(a + 3)^2 + (2a + 1 - 4)^2 = 17</math></p> <p><math>a^2 + 6a + 9 + 4a^2 - 12a + 9 - 17 = 0</math></p> <p><math>5a^2 - 6a + 1 = 0</math></p> <p><math>(5a - 1)(a - 1) = 0</math></p> <p><math>a = \frac{1}{5}</math> or <math>a = 1</math></p> <p><math>\therefore b = \frac{7}{5}</math> or <math>b = 3</math></p> <p><b>OR</b></p> <p><math>a = \frac{b - 1}{2}</math></p> <p><math>17 = (a + 3)^2 + (b - 4)^2</math></p> <p><math>17 = \left( \left( \frac{b - 1}{2} \right) + 3 \right)^2 + (b - 4)^2</math></p> <p><math>17 = \left( \frac{b + 5}{2} \right)^2 + (b - 4)^2</math></p> <p><math>17 = \frac{b^2 + 10b + 25 + 4b^2 - 32b + 64}{4}</math></p> <p><math>68 = 5b^2 - 22b + 89</math></p> <p><math>0 = 5b^2 - 22b + 21</math></p> <p><math>0 = (5b - 7)(b - 3)</math></p> <p><math>\therefore b = \frac{7}{5}</math> or <math>b = 3</math></p>	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p><b>Note:</b> If candidate swops <math>a</math> and <math>b</math> around: <b>max 2 / 6 marks</b></p> </div>	<p>✓ distance formula in terms of <math>a</math> and <math>b</math></p> <p>✓ substitution of <math>b = 2a + 1</math></p> <p>✓ standard form</p> <p>✓ factors or correct substitution into formula</p> <p>✓ values of <math>a</math></p> <p>✓ values of <math>b</math></p> <p style="text-align: right;">(6)</p> <p>✓ <math>a = \frac{b - 1}{2}</math></p> <p>✓ distance formula in terms of <math>a</math> and <math>b</math></p> <p>✓ substitution of <math>a = \frac{b - 1}{2}</math></p> <p>✓ standard form</p> <p>✓ factors or correct substitution into formula</p> <p>✓ values of <math>b</math></p> <p style="text-align: right;">(6)</p> <p style="text-align: right;"><b>[20]</b></p>
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**QUESTION 6**



<p>6.1</p>	<p><math>y = -x + 2</math>  <math>m_{LP} = -1</math>  <math>\therefore m_{LN} = \frac{-1}{-1} = 1</math>  <math>y = x + c</math>  <math>4 = -4 + c</math>  <math>\therefore c = 8</math>  <math>y = x + 8</math></p> <p><b>OR</b></p> <p><math>y - 4 = 1(x + 4)</math>  <math>y = x + 8</math></p>	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p><b>Note:</b>                      If candidate leaves it as <math>y - 4 = x + 4</math>  <b>max 2 / 3 marks</b></p> <p>Answer only: <b>Full marks</b></p> </div>	<p>✓ <math>m_{LP} = -1</math>                      ✓ <math>m_{LN} = 1</math></p> <p>✓ equation (3)</p> <p>✓ <math>m = 1</math>                      ✓ substitution of <math>y - y_1 = m(x - x_1)</math>                      ✓ answer (3)</p>
<p>6.2</p>	<p><math>x + 8 = -x + 2</math>  <math>2x = -6</math>  <math>x = -3</math>  <math>y = -3 + 8</math>  <math>y = 5</math>  <math>L(-3; 5)</math></p> <p style="text-align: center;"><b>OR</b></p>	<p><math>y + x = 2 \dots\dots\dots(1)</math>  <math>y - x = 8 \dots\dots\dots(2)</math>  <math>2y = 10</math>  <math>\therefore y = 5</math>  <math>\therefore x = -3</math>  <math>L(-3; 5)</math></p>	<p>✓ x-value                      ✓ y-value</p> <p>Equations leading to these values must be used (2)</p>

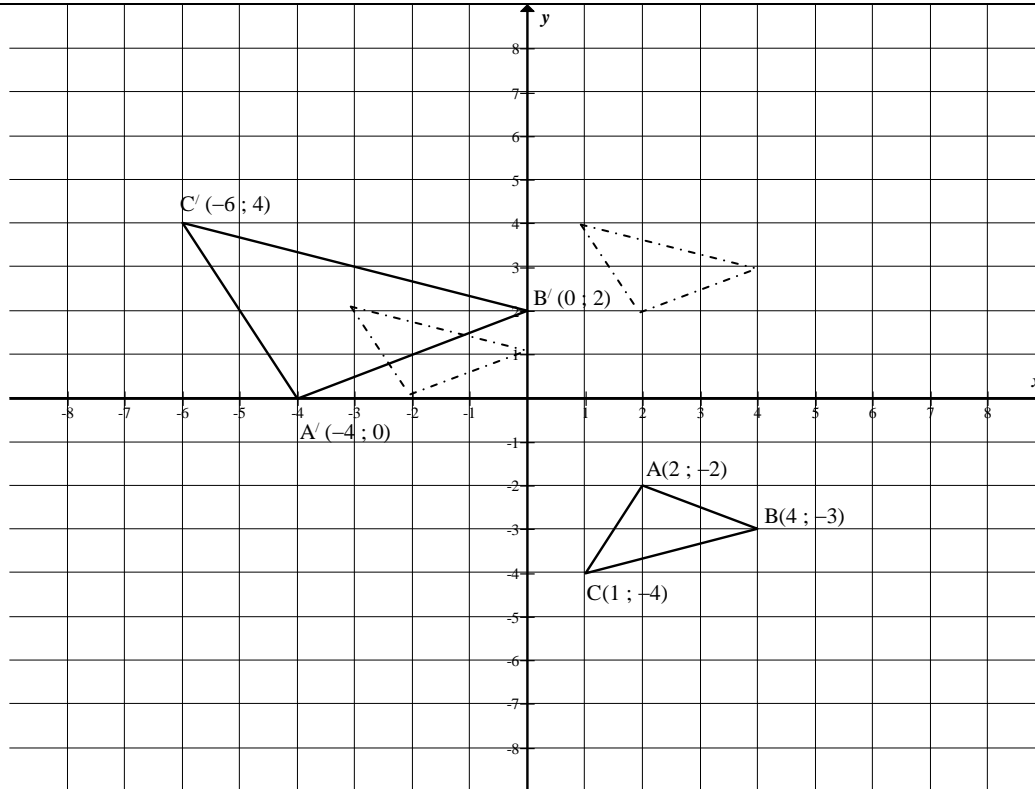
**Note:**  
 No penalty for not leaving in coordinate form

<p>6.3</p>	$(x+4)^2 + (y-4)^2 = r^2$ $(-3+4)^2 + (5-4)^2 = r^2$ $\therefore r^2 = 2$ $(x+4)^2 + (y-4)^2 = 2$ <p>Equation can be left as:</p> $x^2 + 8x + y^2 - 8y + 30 = 0$	<div style="border: 1px solid black; padding: 5px;"> <p><b>Note:</b> If the candidate only uses the distance formula to determine the radius</p> <math display="block">(-3+4)^2 + (5-4)^2 = r^2</math> <math display="block">\therefore r^2 = 2</math> <p>then <b>2 / 3 marks</b></p> </div>	<p>✓ <math>(x+4)^2 + (y-4)^2 = r^2</math> ✓ substitution of <math>(-3 ; 5)</math>  ✓ <math>r^2 = 2</math> (3)</p>
<p>6.4</p>	<p>Let N(x, y). Since M(-4 ; 4) is the midpoint of LN and L(-3 ; 5)</p> $\frac{x-3}{2} = -4; \quad \frac{y+5}{2} = 4$ $\therefore x = -5; \quad y = 3$ <p><b>OR</b></p> $y = x + 8$ $(x+4)^2 + (y-6)^2 = 2$ $(x+4)^2 + (x+8-4)^2 - 2 = 0$ $x^2 + 8x + 16 + x^2 + 8x + 16 - 2 = 0$ $2x^2 + 16x + 30 = 0$ $x^2 + 8x + 15 = 0$ $(x+5)(x+3) = 0$ $x = -3 \quad \text{or} \quad x = -5$ $y = 5 \quad \quad y = 3$ $\therefore N(-5 ; 3)$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p><b>Note:</b> Answer only: <b>Full marks</b></p> </div>	<p>✓ using the fact that M is the midpoint of LN ✓ <math>x = -5</math> ✓ <math>y = 3</math> (3)</p> <p>✓ <math>(x+4)^2 + (x+8-4)^2 - 2 = 0</math></p> <p>✓ <math>x = -5</math> ✓ <math>y = 3</math> (3)</p>
<p>6.5</p>	$m_{NQ} = -1$ $y = -x + c$ $3 = -(-5) + c$ $c = -2$ $y = -x - 2$ <p><b>OR</b></p> $m_{NQ} = -1$ $y - 3 = -(x + 5)$ $y = -x - 2$ <p><b>OR</b></p> <p>Equation of LP is <math>x + y = 2</math> NQ <math>\parallel</math> LP <math>\therefore</math> equation of NQ is <math>x + y = k</math> for some <math>k \in R</math> But N(-5 ; 3) lies on NQ <math>\therefore x + y = -5 + 3 = -2</math></p>	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p><b>Note:</b> Answer only: <b>Full marks</b></p> </div>	<p>✓ gradient ✓ substitution of <math>(-5 ; 3)</math> into <math>y = mx + c</math> ✓ <math>c = -2</math> (3)</p> <p>✓ gradient ✓ substitution of <math>(-5 ; 3)</math> into <math>y - y_1 = m(x - x_1)</math> ✓ equation (3)</p> <p>✓ <math>x + y = k</math> ✓ substitution of <math>(-5 ; 3)</math> ✓ equation (3)</p>

	<p><b>OR</b> NQ is a reflection of LP (<math>y + x = 2</math>) in the line <math>y = x</math> <math>\therefore</math> equation of NQ is <math>x + y = -2</math></p>	
<p>6.6</p>	<p>Let new radius of circle be R and centre be M'.  <math>M'(-4 + 6; 4)</math>  <math>= (2; 4)</math>  <math>R = 2r</math>  <math>R^2 = 4r^2</math>  <math>= 4(2)</math>  <math>= 8</math>  <math>\therefore (x - 2)^2 + (y - 4)^2 = (2\sqrt{2})^2</math>  <math>\therefore (x - 2)^2 + (y - 4)^2 = 8</math></p> <p><b>OR</b></p> <p>Let R = new radius of circle  <math>R^2 = (2r)^2 = 4(2) = 8</math>  <math>(x - 6 + 4)^2 + (y - 4)^2 = 8</math>  <math>\therefore (x - 2)^2 + (y - 4)^2 = 8</math></p>	<p><math>\checkmark M'(2; 4)</math>  <math>\checkmark r = 2\sqrt{2}</math>  <math>\checkmark</math> equation  <span style="float: right;">(3)</span></p> <p><math>\checkmark (x - 2)^2</math>  <math>\checkmark (y - 4)^2</math>  <math>\checkmark 8</math> or <math>(2\sqrt{2})^2</math>  <span style="float: right;">(3)</span></p> <p style="text-align: right;"><b>[17]</b></p>

**QUESTION 7**

7.1



2 marks for each diagram of the transformation (6)

**OR**

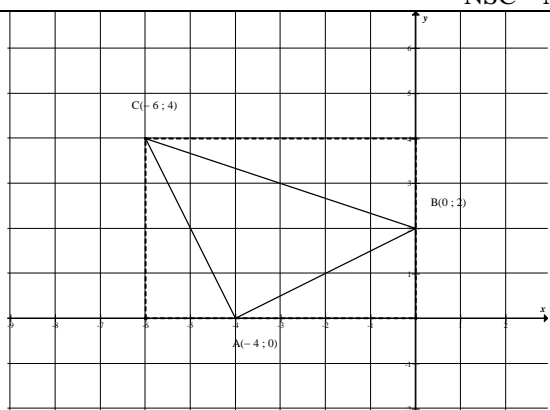
If candidate works out the general rule first  
 $(x ; y) \rightarrow (2x - 8; -2y - 4)$   
 ✓  $A'(-4 ; 0)$   
 ✓  $B'(0 ; 2)$   
 ✓  $C'(-6 ; 4)$   
 1 mark per each correct point plotted and joined (6)

- Note:**
- If the candidate only draws the correct triangle with labels, **full marks**
  - If they plot the points correctly and do not draw the triangle, **max 5 / 6 marks**
  - In the 3 sketches, if one vertex of the three is wrong, then 1 / 2 marks for the incorrect sketch, then CA applies.
  - If they write down the points and do not plot the points and draw the triangle **max 3 / 6 marks**
  - If the vertices are correct but not labelled and the points are joined **max 5 / 6 marks**
  - If the vertices are correct, not labelled and not joined **max 4 / 6 marks**
  - If a candidate finds a formula first and gets it wrong
    - Max 1 mark for the formula
    - Max 2 marks for the calculation of  $A', B', C'$  coordinates (CA)
    - 1 mark for plotting 3 vertices
    - 1 mark for completing the triangle and labelling



<p>7.2</p> <p><math>(x ; y) \rightarrow (x ; -y)</math>  <math>(x ; -y) \rightarrow (x - 4 ; -y - 2)</math>  <math>(x - 4 ; -y - 2) \rightarrow (2x - 8 ; -2y - 4)</math></p> <p><b>OR</b></p> <p><math>(x ; y) \rightarrow (2x - 8 ; -2y - 4)</math></p> <div style="border: 1px solid black; padding: 10px; margin-top: 10px;"> <p><b>Note:</b></p> <ul style="list-style-type: none"> <li>• If the candidate writes <math>(x ; y) \rightarrow (x ; -y)</math>  <math>(x ; y) \rightarrow (x - 4 ; y - 2)</math>  <math>(x ; y) \rightarrow (2x ; 2y)</math></li> </ul> <p>then <b>2 / 4 marks</b></p> <ul style="list-style-type: none"> <li>• If the candidate writes <math>(x ; y) \rightarrow (x ; -y)</math>  <math>(x ; y) \rightarrow (x - 4 ; -y - 2)</math>  <math>(x ; y) \rightarrow (2x - 8 ; -2y - 4)</math></li> </ul> <p>then <b>4 / 4 marks</b></p> <ul style="list-style-type: none"> <li>• If candidate writes <math>(x ; y) \rightarrow 2(x - 4 ; -y - 2)</math> then <b>3 / 4 marks</b></li> </ul> </div>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p><b>Note:</b> Answer only: Full marks</p> </div> <p>✓  <math>(x ; y) \rightarrow (x ; -y)</math></p> <p>✓ <math>x - 4</math>          ✓ <math>-y - 2</math>          ✓  <math>(2x - 8 ; -2y - 4)</math></p> <p>(4)</p>
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<p>7.3</p> <p>Area <math>\Delta ABC</math> = area of rectangle – sum of 3 triangle areas</p> $= 6 - \left(1 + \frac{3}{2} + 1\right)$ $= \frac{5}{2}$ <p>Area <math>\Delta A'B'C'</math> = <math>2^2 \left(\frac{5}{2}\right)</math></p> $= 10 \text{ units}^2$ <p><b>OR</b></p>	<p>✓ <math>6 - \left(1 + \frac{3}{2} + 1\right)</math></p> <p>✓ <math>\frac{5}{2}</math></p> <p>✓✓ 10</p> <p>(4)</p>
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$$\begin{aligned} \text{Area } \Delta A'B'C' &= 24 - \left( \left( \frac{1}{2} \cdot 6 \cdot 2 \right) + \frac{1}{2} (4)(2) + \frac{1}{2} (2)(4) \right) \\ &= 24 - 6 - 4 - 4 \\ &= 10 \text{ units}^2 \end{aligned}$$

**OR**

$$m_{AC} = 2 \text{ and } m_{AB} = -\frac{1}{2} \therefore \text{product} = -1$$

$$\therefore \hat{CAB} = 90^\circ$$

$$AB = \sqrt{5}$$

$$AC = \sqrt{5}$$

$$\therefore \text{Area } \Delta ABC \text{ is } \frac{1}{2} (\sqrt{5})^2 = \frac{5}{2}$$

$$\therefore \text{Area } \Delta A'B'C' \text{ is } 4 \times \frac{5}{2} = 10 \text{ square units}$$

**OR**

$$m_{A'C'} = -2 \text{ and } m_{A'B'} = \frac{1}{2} \therefore \text{product} = -1$$

$$\therefore \hat{C'A'B'} = 90^\circ$$

$$A'B' = \sqrt{20}$$

$$A'C' = \sqrt{20}$$

$$\therefore \text{Area } \Delta A'B'C' \text{ is } \frac{1}{2} (\sqrt{20})^2 = 10 \text{ square units}$$

**OR**

$$AB = \sqrt{5}$$

$$AC = \sqrt{5}$$

$$BC^2 = 10$$

$$BC = \sqrt{10}$$

$$\perp \text{height} = \sqrt{\frac{10}{4}}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \cdot \sqrt{\frac{10}{4}} \cdot \sqrt{10} = \frac{5}{2}$$

$$\text{Area } \Delta A'B'C' = 4 \times \frac{5}{2} = 10 \text{ square units}$$

$$\checkmark \left( \left( \frac{1}{2} \cdot 6 \cdot 2 \right) + \frac{1}{2} (4)(2) + \frac{1}{2} (2)(4) \right)$$

$$\checkmark 24$$

$$\checkmark \checkmark 10$$

(4)

$$\checkmark AB = \sqrt{5} \text{ and}$$

$$AC = \sqrt{5}$$

$$\checkmark \frac{5}{2}$$

$$\checkmark \checkmark 10$$

(4)

$$\checkmark A'B' = \sqrt{20}$$

$$\checkmark A'C' = \sqrt{20}$$

$$\checkmark \checkmark 10$$

(4)

$$\checkmark AB = \sqrt{5} \text{ and}$$

$$AC = \sqrt{5}$$

$$\checkmark \frac{5}{2}$$

$$\checkmark \checkmark \text{ answer}$$

(4)

**OR**

$$AB = \sqrt{5}$$

$$AC = \sqrt{5}$$

$$BC^2 = 10$$

$$BC = \sqrt{10}$$

$$AC^2 + AB^2 = BC^2$$

$$\therefore \hat{CAB} = 90^\circ$$

$$\text{Area } \Delta ABC = = \frac{1}{2} \cdot (\sqrt{5})^2 = \frac{5}{2}$$

$$\text{Area } \Delta A'B'C' = 4 \times \frac{5}{2} = 10 \text{ square units}$$

**OR**

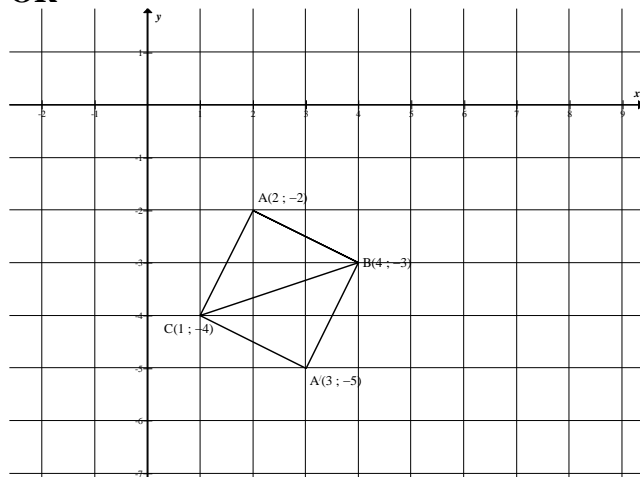
$$\text{Area } \Delta ABC = = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} (\sqrt{5})(\sqrt{5}) \sin 90$$

$$= \frac{5}{2}$$

$$\text{Area } \Delta A'B'C' = 4 \times \frac{5}{2} = 10 \text{ square units}$$

**OR**



Reflect  $\Delta ABC$  about  $CB$  and get the square  $ABA''C$  of side  $\sqrt{5}$

$$\text{Area square} = (\sqrt{5})(\sqrt{5}) = 5$$

$$\text{Area } \Delta ABC = \frac{5}{2}$$

$$\text{Area } \Delta A'B'C' = 4 \times \frac{5}{2} = 10 \text{ square units}$$

**OR**

$$\checkmark AB = \sqrt{5} \text{ and } AC = \sqrt{5}$$

$$\checkmark \frac{5}{2}$$

$$\checkmark \checkmark \text{ answer}$$

(4)

$$\checkmark AB = \sqrt{5} \text{ and } AC = \sqrt{5}$$

$$\checkmark \frac{5}{2}$$

$$\checkmark \checkmark 10$$

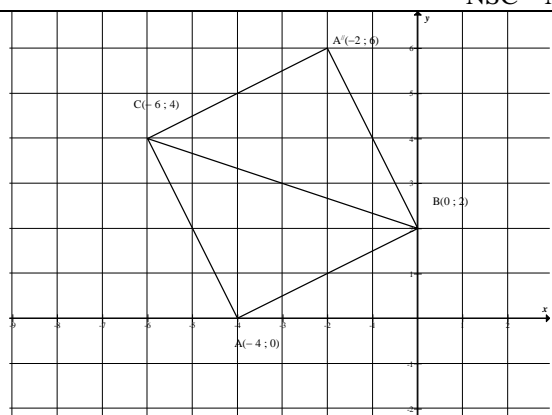
(4)

$$\checkmark AB = \sqrt{5} \text{ and } AC = \sqrt{5}$$

$$\checkmark \text{ reflection to get square}$$

$$\checkmark \checkmark 10$$

(4)



Reflect  $\Delta A'B'C'$  about  $C'B'$  and get the square  $A'B'A''C'$  of side  $\sqrt{20}$

$$\text{Area square} = (\sqrt{20})(\sqrt{20}) = 20$$

$$\text{Area } \Delta A'B'C' = \frac{1}{2} \times 20 = 10 \text{ square units}$$

- ✓  $A'B' = \sqrt{20}$  and  $A'C' = \sqrt{20}$
- ✓ reflection to get square
- ✓✓ 10

(4)  
[14]

**QUESTION 8**

<p>8.1</p> $  \begin{aligned}  x' &= x \cos \alpha - y \sin \alpha \\  &= 2 \cos 75^\circ - 4 \sin 75^\circ \\  &= 2 \cos(30^\circ + 45^\circ) - 4 \sin(30^\circ + 45^\circ) \\  &= 2 \cos 30^\circ \cos 45^\circ - 2 \sin 30^\circ \sin 45^\circ - 4 \sin 30^\circ \cos 45^\circ - 4 \cos 30^\circ \sin 45^\circ \\  &= 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - 4 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\  &= \frac{2\sqrt{6} - 2\sqrt{2} - 4\sqrt{2} - 4\sqrt{6}}{4} \\  &= \frac{-2\sqrt{2} - 6\sqrt{6}}{4} \\  &= \frac{-\sqrt{6} - 3\sqrt{2}}{2} \quad \text{or} \quad -\frac{1}{\sqrt{2}}(3 + \sqrt{3})  \end{aligned}  $ <p><b>OR</b></p> $  \begin{aligned}  x' &= x \cos \alpha + y \sin \alpha \\  &= 2 \cos(-75^\circ) + 4 \sin(-75^\circ) \\  &= 2 \cos(75^\circ) - 4 \sin(75^\circ) \\  &= 2 \cos(30^\circ + 45^\circ) - 4 \sin(30^\circ + 45^\circ) \\  &= 2 \cos 30^\circ \cos 45^\circ - 2 \sin 30^\circ \sin 45^\circ - 4 \sin 30^\circ \cos 45^\circ - 4 \cos 30^\circ \sin 45^\circ \\  &= 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - 4 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\  &= \frac{2\sqrt{6} - 2\sqrt{2} - 4\sqrt{2} - 4\sqrt{6}}{4} \\  &= \frac{-2\sqrt{2} - 6\sqrt{6}}{4} \\  &= \frac{-\sqrt{6} - 3\sqrt{2}}{2} \quad \text{or} \quad -\frac{1}{\sqrt{2}}(3 + \sqrt{3})  \end{aligned}  $ <p><b>OR</b></p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p><b>Note:</b> If the candidate uses a calculator i.e. gives a decimal answer <b>max 5 / 6 marks</b></p> <p><b>Incorrect formula:</b> <b>max 5 / 6 marks</b></p> </div> <p>✓ substitution using anti clockwise formula</p> <p>✓ 75 = 30 + 45</p> <p>✓ cos expansion</p> <p>✓ sin expansion</p> <p>✓ substitution of special angles</p> <p>✓ simplified answer for <math>x'</math></p> <p style="text-align: right;">(6)</p> <p>✓ substitution using clockwise formula</p> <p>✓ 75 = 30 + 45</p> <p>✓ cos expansion</p> <p>✓ sin expansion</p> <p>✓ substitution of special angles</p> <p>✓ simplified answer for <math>x'</math></p> <p style="text-align: right;">(6)</p>
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<p>First compute</p> $\begin{aligned}\cos 75^\circ &= \cos(30^\circ + 45^\circ) \\ &= \cos 30^\circ \cdot \cos 45^\circ - \sin 30^\circ \cdot \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$ <p>and</p> $\begin{aligned}\sin 75^\circ &= \sin(30^\circ + 45^\circ) \\ &= \sin 30^\circ \cdot \cos 45^\circ + \cos 30^\circ \cdot \sin 45^\circ \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2} + \sqrt{6}}{4}\end{aligned}$ $\begin{aligned}x' &= 2 \cos 75^\circ - 4 \sin 75^\circ \\ &= 2 \left( \frac{\sqrt{6} - \sqrt{2}}{4} \right) - 4 \left( \frac{\sqrt{2} + \sqrt{6}}{4} \right) \\ &= \frac{-2\sqrt{6} - 6\sqrt{2}}{4} \\ &= \frac{-\sqrt{6} - 3\sqrt{2}}{2}\end{aligned}$	<ul style="list-style-type: none"> <li>✓ <math>75 = 30 + 45</math></li> <li>✓ cos expansion</li>   <li>✓ substitution of special angles in the first expansion</li>   <li>✓ sin expansion</li>   <li>✓ substitution</li>   <li>✓ simplified answer for <math>x'</math></li> </ul> <p style="text-align: right;">(6)</p>
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<p>8.2</p> $x' = x \cos \beta - y \sin \beta$ $3 \cos \beta - \sin \beta = \frac{3 - \sqrt{3}}{2} \quad \dots (1)$ $y' = y \cos \beta + x \sin \beta$ $\cos \beta + 3 \sin \beta = \frac{1 + 3\sqrt{3}}{2} \quad \dots (1^*)$ <p>(1)×3 + (1)*:</p> $10 \cos \beta = 3 \left( \frac{3 - \sqrt{3}}{2} \right) + \frac{1 + 3\sqrt{3}}{2}$ $= \frac{1}{2} (9 - 3\sqrt{3} + 1 + 3\sqrt{3})$ $= 10 \left( \frac{1}{2} \right)$ <p>∴ <math>\cos \beta = \frac{1}{2}</math> ∴ <math>\beta = 60^\circ</math></p> <p><b>OR</b></p> <p>(1) – 3(1)*:</p> $-10 \sin \beta = 3 \left( \frac{1}{2} \right) - \left( \frac{3 - \sqrt{3}}{2} \right)$ $= \frac{3 - \sqrt{3} - 3(1 + 3\sqrt{3})}{2}$ $= \frac{-10\sqrt{3}}{2}$ <p>∴ <math>\sin \beta = \frac{\sqrt{3}}{2}</math> ∴ <math>\beta = 60^\circ</math></p> <p><b>OR</b></p> $3 \cos \beta - \sin \beta = \frac{3 - \sqrt{3}}{2} \quad \dots (1)$ $\cos \beta = \frac{1 + 3\sqrt{3}}{2} - 3 \sin \beta \quad \dots (2)$ <p>Substitute (2) into (1)</p> $3 \left( \frac{1 + 3\sqrt{3}}{2} - 3 \sin \beta \right) - \sin \beta = \frac{3 - \sqrt{3}}{2}$ $\frac{3 + 9\sqrt{3}}{2} - 9 \sin \beta - \sin \beta = \frac{3 - \sqrt{3}}{2}$ $-10 \sin \beta = \frac{3 - \sqrt{3} - 3 - 9\sqrt{3}}{2}$ $-10 \sin \beta = \frac{-10\sqrt{3}}{2}$ $\sin \beta = \frac{\sqrt{3}}{2}$ $\beta = 60^\circ$	<p>✓ substitution into <math>x'</math></p> <p>✓ substitution into <math>y'</math></p> <p>✓ simplification</p> <p>✓ solving simultaneously</p> <p>✓ <math>\frac{\sqrt{3}}{2}</math> or <math>\frac{1}{2}</math></p> <p>✓ answer</p> <p style="text-align: right;">(6)</p> <p>✓ equation (1)</p> <p>✓ equation (2)</p> <p>✓ substitution</p> <p>✓ simplification</p> <p>✓ <math>\sin \beta = \frac{\sqrt{3}}{2}</math></p> <p>✓ answer</p> <p style="text-align: right;">(6)</p>
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**OR**

$$3 \cos \beta - \sin \beta = \frac{3 - \sqrt{3}}{2}$$

$$\text{and } \cos \beta + 3 \sin \beta = \frac{1 + 3\sqrt{3}}{2}$$

Try  $\beta = 60^\circ$

$$3 \cos \beta - \sin \beta = 3 \left( \frac{1}{2} \right) - 3 \left( \frac{\sqrt{3}}{2} \right) = \frac{3 - \sqrt{3}}{2}$$

$$\cos \beta + 3 \sin \beta = \frac{1}{2} + 3 \left( \frac{\sqrt{3}}{2} \right) = \frac{1 + 3\sqrt{3}}{2}$$

$$\therefore \beta = 60^\circ$$

**Note:**

Answer only:  
**max 2 / 6 marks**

✓✓  $\beta = 60^\circ$

✓ substitution  
✓ simplification

✓ substitution  
✓ simplification

(6)

**OR**

$$\tan \alpha = \frac{1}{3}$$

$$\alpha = 18,43^\circ$$

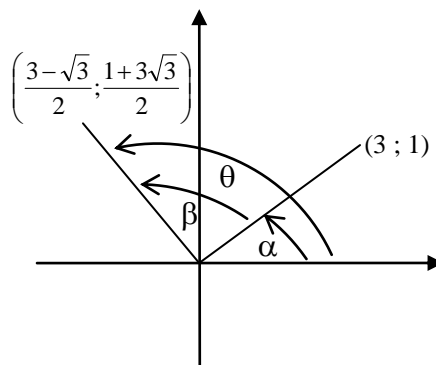
$$\tan \theta = \frac{\frac{1 + 3\sqrt{3}}{2}}{\frac{3 - \sqrt{3}}{2}}$$

$$= \frac{1 + 3\sqrt{3}}{3 - \sqrt{3}}$$

$$\theta = 78,43^\circ$$

$$\beta = 78,43^\circ - 18,43^\circ$$

$$= 60^\circ$$



✓  $\tan \alpha = \frac{1}{3}$   
✓  $\alpha = 18,43^\circ$

✓  $\tan \theta = \frac{1 + 3\sqrt{3}}{3 - \sqrt{3}}$

✓  $\theta = 78,43^\circ$

✓ simplification  
✓ answer

$$\left( \frac{3 - \sqrt{3}}{2}; \frac{1 + 3\sqrt{3}}{2} \right)$$

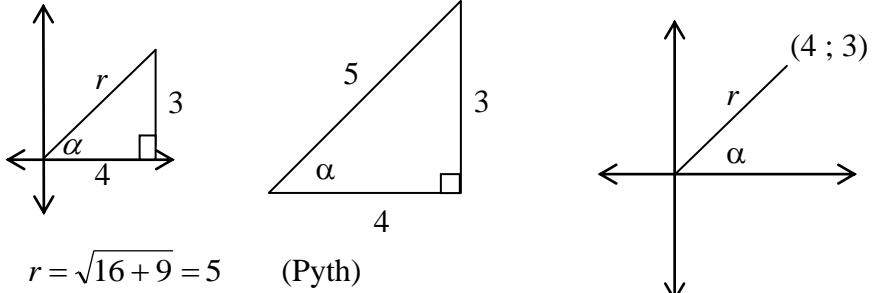
(6)

**OR**



$x' = x \cos \beta - y \sin \beta$ $\frac{3 - \sqrt{3}}{2} \cos \beta - \frac{1 + 3\sqrt{3}}{2} \sin \beta = 3$ $3 \cos \beta - \sqrt{3} \cos \beta - \sin \beta - 3\sqrt{3} \sin \beta = 6$ $\frac{1 + 3\sqrt{3}}{2} \cos \beta + \frac{3 - \sqrt{3}}{2} \sin \beta = 1$ $\cos \beta + 3\sqrt{3} \cos \beta + 3 \sin \beta - \sqrt{3} \sin \beta = 2$ $3 \cos \beta + 9\sqrt{3} \cos \beta + 9 \sin \beta - 3\sqrt{3} \sin \beta = 6$ $3 \cos \beta - \sqrt{3} \cos \beta - \sin \beta - 3\sqrt{3} \sin \beta = 6$ $10\sqrt{3} \cos \beta + 10 \sin \beta = 0$ $\sin \beta = -\sqrt{3} \cos \beta$ $\cos \beta + 3\sqrt{3} \cos \beta + 3(-\sqrt{3} \cos \beta) - \sqrt{3}(-\sqrt{3} \cos \beta) = 2$ $4 \cos \beta = 2$ $\cos \beta = \frac{1}{2}$ $\beta = 60^\circ$	<p>✓ substitution</p> <p>✓ substitution</p> <p>✓ simplification</p> <p>✓ <math>\sin \beta = -\sqrt{3} \cos \beta</math></p> <p>✓ <math>\cos \beta = \frac{1}{2}</math></p> <p>✓ answer</p> <p style="text-align: right;">(6)</p> <p style="text-align: right;"><b>[12]</b></p>
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**QUESTION 9**

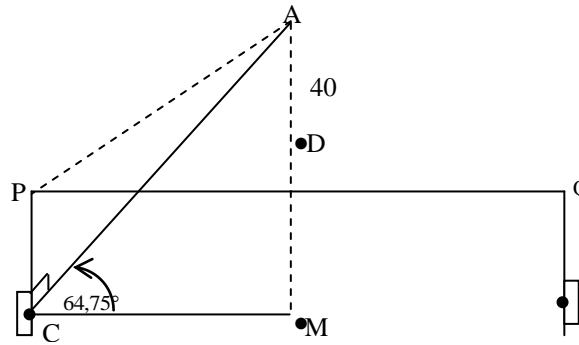
<p>9.1</p>	 <p><math>r = \sqrt{16 + 9} = 5</math></p> <p><math>\sin \alpha = \frac{3}{5}</math></p> <p>Accept : 0,6</p>	<p>✓ any one of the diagram</p> <p>✓ value of <math>r</math></p> <p>✓ answer (3)</p>
<p>9.2</p>	$\begin{aligned} &\cos^2(90^\circ - \alpha) - 1 \\ &= \sin^2 \alpha - 1 \\ &= \left(\frac{3}{5}\right)^2 - \frac{25}{25} \\ &= \frac{-16}{25} \\ &= -0,64 \end{aligned}$ <p style="text-align: center;"><b>OR</b></p> $\begin{aligned} &\cos^2(90^\circ - \alpha) - 1 \\ &= \sin^2 \alpha - (\sin^2 \alpha + \cos^2 \alpha) \\ &= -\cos^2 \alpha \\ &= -\left(\frac{4}{5}\right)^2 \\ &= \frac{-16}{25} \\ &= -0,64 \end{aligned}$	<p>✓ <math>\cos(90^\circ - \alpha) = \sin \alpha</math></p> <p>✓ substitution of <math>\sin \alpha = \frac{3}{5}</math> (2)</p>
<p>9.3</p>	$\begin{aligned} &1 - \sin 2\alpha \\ &= 1 - 2 \sin \alpha \cos \alpha \\ &= 1 - 2 \left(\frac{3}{5}\right) \left(\frac{4}{5}\right) \\ &= 1 - \frac{24}{25} \\ &= \frac{1}{25} \end{aligned}$ <p style="text-align: center;"><b>OR</b></p> $\begin{aligned} &1 - \sin 2\alpha \\ &= \sin^2 \alpha - 2 \sin \alpha \cos \alpha + \cos^2 \alpha \\ &= (\sin \alpha - \cos \alpha)^2 \\ &= \left(\left(\frac{3}{5}\right) - \left(\frac{4}{5}\right)\right)^2 \\ &= \left(-\frac{1}{5}\right)^2 \\ &= \frac{1}{25} \end{aligned}$	<p>✓ <math>\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha</math></p> <p>✓ <math>\frac{24}{25}</math></p> <p>✓ answer (3)</p> <p>✓ <math>\sin^2 \alpha + \cos^2 \alpha = 1</math></p> <p>✓ <math>\left(\left(\frac{3}{5}\right) - \left(\frac{4}{5}\right)\right)^2</math></p> <p>✓ answer (3)</p> <p style="text-align: right;"><b>[8]</b></p>

**QUESTION 10**

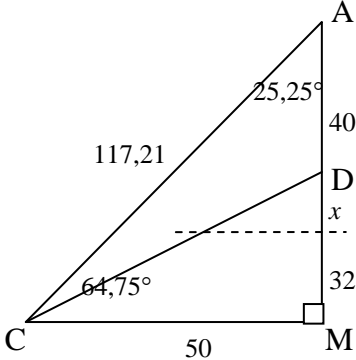
<p>10.1</p>	$\frac{\sin(90^\circ + \theta) + \cos(180^\circ + \theta) \sin(-\theta)}{\sin 180^\circ - \tan 135^\circ}$ $= \frac{\cos \theta + (-\cos \theta)(-\sin \theta)}{0 + 1}$ $= \cos \theta + \cos \theta \cdot \sin \theta$ $= \cos \theta(1 + \sin \theta)$	<p>✓ <math>\cos \theta</math>                  ✓ <math>-\cos \theta</math>                  ✓ <math>-\sin \theta</math>                  ✓ <math>0 + 1</math>                  ✓ answer</p> <p>(5)</p>
<p>10.2</p>	$\frac{4 \sin A \cos A \cos 2A \cdot \sin 15^\circ}{\sin 2A(1 - 2 \sin^2 A)}$ $= \frac{4 \sin A \cos A \cos 2A \cdot \sin 15^\circ}{2 \sin A \cos A(1 - 2 \sin^2 A)}$ $= \frac{2 \cos 2A \cdot \sin 15^\circ}{\cos 2A}$ $= 2 \sin 15^\circ$ $= 2 \sin(45^\circ - 30^\circ)$ $= 2[\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ]$ $= 2 \left[ \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \right]$ $= 2 \left[ \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \right]$ $= \frac{\sqrt{6} - \sqrt{2}}{2}$ <p><b>OR</b></p> <p>Left Hand Side = <math>\frac{4 \sin A \cos A \cos 2A \cdot \sin 15^\circ}{2 \sin A \cos A(1 - 2 \sin^2 A)}</math></p> $= \frac{2 \cos 2A \cdot \sin 15^\circ}{\cos 2A}$ $= 2 \sin 15^\circ$ $= 2 \sin(60^\circ - 45^\circ)$ $= 2[\sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ]$ $= 2 \left[ \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \right]$ $= 2 \left[ \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \right]$ $= \frac{\sqrt{6} - \sqrt{2}}{2} = \text{RHS}$	<p>✓ <math>2 \sin A \cos A</math>                  ✓ <math>1 - 2 \sin^2 A = \cos 2A</math>                  ✓ <math>2 \sin 15^\circ</math>                  ✓ <math>15 = 45 - 30</math> or  <math>15 = 60 - 45</math>                  ✓ substitution                  ✓ <math>2 \left[ \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \right]</math></p> <p>(6)</p> <p>✓ <math>2 \sin A \cos A</math>                  ✓ <math>1 - 2 \sin^2 A = \cos 2A</math>                  ✓ <math>2 \sin 15^\circ</math>                  ✓ <math>15 = 45 - 30</math> or  <math>15 = 60 - 45</math>                  ✓ substitution                  ✓ <math>2 \left[ \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \right]</math></p> <p>(6)</p>

	<p><b>OR</b></p> $\text{Left Hand Side} = \frac{4 \sin A \cos A \cos 2A \cdot \sin 15^\circ}{2 \sin A \cos A (1 - 2 \sin^2 A)}$ $= \frac{2 \sin 2A \cos 2A \cdot \sin 15^\circ}{\sin 2A \cos 2A}$ $= 2 \sin 15^\circ$ $= 2 \sin(45^\circ - 30^\circ)$ $= 2[\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ]$ $= 2 \left[ \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \right]$ $= 2 \left[ \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \right]$ $= \frac{\sqrt{6} - \sqrt{2}}{2} = \text{RHS}$	
<p>10.3</p>	$6 \cos x - 5 = \frac{4}{\cos x}$ $6 \cos^2 x - 5 \cos x = 4$ $6 \cos^2 x - 5 \cos x - 4 = 0$ $(3 \cos x - 4)(2 \cos x + 1) = 0$ $\cos x = \frac{4}{3} \quad \text{or} \quad \cos x = \frac{-1}{2}$ <p>no solution    or    <math>x = 120^\circ + k \cdot 360^\circ, k \in \mathbb{Z}</math></p> <p style="text-align: center;">or</p> $x = 240^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$ <p style="text-align: center;">Alternative solution for <math>\cos x = \frac{-1}{2}</math></p> $x = k \cdot 360^\circ \pm 120^\circ, k \in \mathbb{Z}$ <div style="border: 1px solid black; padding: 5px; width: fit-content;"> <p><b>Note:</b> If candidate puts <math>\pm k \cdot 360</math> then <math>k \in \mathbb{N}_0</math></p> </div>	<ul style="list-style-type: none"> <li>✓ standard form</li> <li>✓ factors</li> <li>✓ both equations</li> <li>✓ <math>240^\circ + k \cdot 360^\circ</math></li> <li>✓ <math>120^\circ + k \cdot 360^\circ</math></li> <li>✓ <math>k \in \mathbb{Z}</math></li> </ul> <p style="text-align: right;">(6)</p> <p style="text-align: right;"><b>[17]</b></p>

**QUESTION 11**

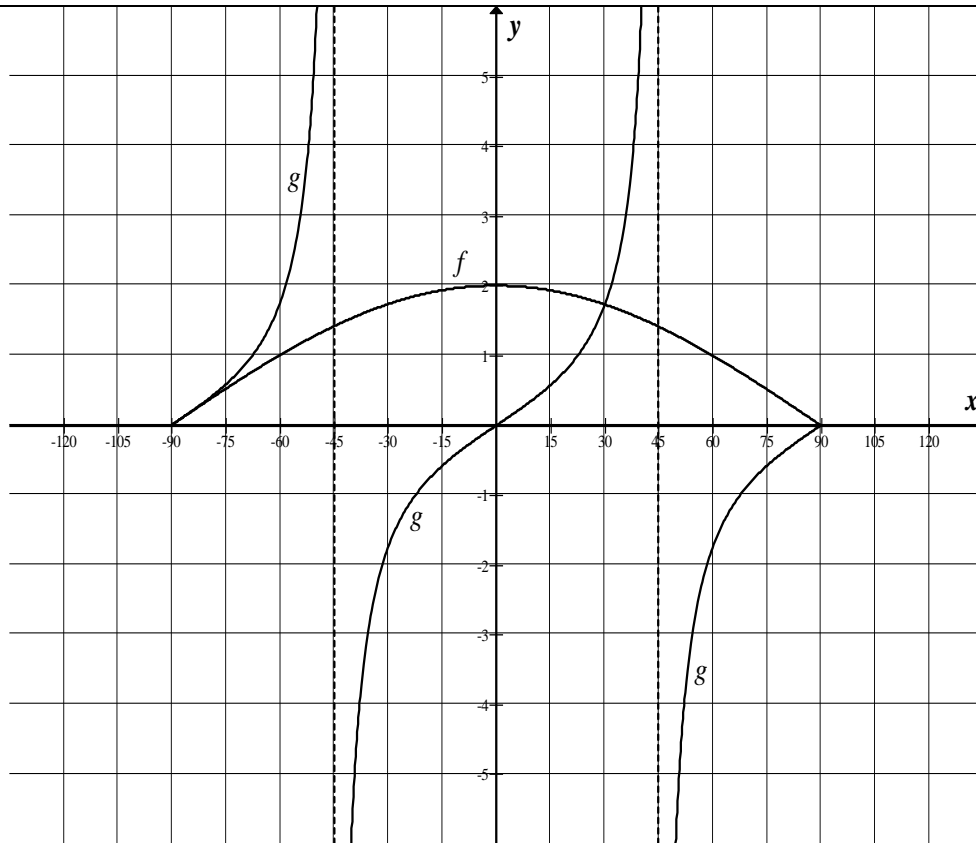


<p>11.1</p>	$\cos 64,75^\circ = \frac{50}{AC}$ $AC = \frac{50}{\cos 64,75^\circ}$ $= 117,21 \text{ m}$ <p><b>OR</b></p> $AC = \frac{50}{\cos 64,75^\circ}$ $\therefore AC = 117,2144026 \text{ m}$ $\therefore AC = 117,21 \text{ m}$ <p><b>OR</b></p> $\frac{50}{\sin 25,25^\circ} = \frac{AC}{\sin 90^\circ}$ $\therefore AC = \frac{50 \sin 90^\circ}{\sin 25,25^\circ}$ $\therefore AC = 117,21 \text{ m}$	<ul style="list-style-type: none"> <li>✓ substitution in ratio</li> <li>✓ AC subject of the formula</li> <li>✓ answer (3)</li> </ul> <ul style="list-style-type: none"> <li>✓ substitution into ratio</li> <li>✓ AC subject of the formula</li> <li>✓ answer (3)</li> </ul> <ul style="list-style-type: none"> <li>✓ substitution into sine rule</li> <li>✓ AC subject of the formula</li> <li>✓ answer (3)</li> </ul>
<p>11.2</p>	<p>PC is given to be <math>\frac{1}{2}(64) = 32 \text{ m}</math></p> $\tan \hat{PAC} = \frac{32}{117,21}$ $\theta = 15,27^\circ \text{ (15,27042173...)}$ <p><b>Note:</b> If the candidate takes the unrounded answer for AC, then the answer is <math>15,27^\circ</math> (15,26987495...)</p>	<ul style="list-style-type: none"> <li>✓ <math>\tan \hat{PAC}</math></li> <li>✓ <math>\frac{32}{117,21}</math></li> <li>✓ answer (3)</li> </ul>

<p>11.3</p>	<p> <math>CD^2 = 117,21^2 + 40^2 - 2(117,21)(40) \cos 25,25</math>  <math>= 6857,289092</math>  <math>\therefore CD = 82,81 \text{ m}</math> </p> <p><b>OR</b></p>  <p> <math>AM = AC \sin 64,75^\circ</math> <b>OR</b> <math>AM = CM \tan 64,75^\circ</math> <b>OR</b> <math>AM = AC \cos 25,25^\circ</math>  <math>= 106,0111876</math>                      <math>= 50 \tan 64,75^\circ</math>                      <math>= 117,21 \cdot \cos 25,25^\circ</math>  <math>= 106,01</math>                                      <math>= 106,01</math>                                      <math>= 106,01</math> </p> <p> <math>DM = 106,01 - 40</math>  <math>= 66,01</math> </p> <p> <math>CD^2 = CM^2 + DM^2</math>  <math>= (50)^2 + (66,01)^2</math>  <math>= 6857,3201</math>  <math>CD = 82,81 \text{ metres}</math> </p> <p><b>OR</b></p> <p> <math>AM = AC \sin 64,75^\circ</math> <b>OR</b> <math>AM = CM \tan 64,75^\circ</math> <b>OR</b> <math>AM = AC \cos 25,25^\circ</math>  <math>= 106,0111876</math>                      <math>= 50 \tan 64,75^\circ</math>                      <math>= 117,21 \cdot \cos 25,25^\circ</math>  <math>= 106,01</math>                                      <math>= 106,01</math>                                      <math>= 106,01</math> </p> <p> <math>DM = 106,01 - 40</math>  <math>= 66,01</math> </p> <p> <math>DC^2 = (50)^2 + (66,01)^2 - 2(50)(66,01) \cdot \cos 90^\circ</math>  <math>= 6857,3201</math>  <math>CD = 82,81 \text{ metres}</math> </p> <p><b>OR</b></p> <p> <math>\sin 64,75^\circ = \frac{40 + x + 32}{117,21}</math>  <math>x = 34,01</math> </p> <p> <math>CD^2 = CM^2 + DM^2</math>  <math>= (50)^2 + (32 + 34,01)^2</math>  <math>= 6857,3201</math>  <math>CD = 82,81 \text{ metres}</math> </p>	<p>                 ✓ cos rule                  ✓✓ substitution                  ✓ answer                  (4)             </p> <p>                 ✓ AM = 106,01                  ✓ DM = 66,01                  ✓ Pythagoras                  ✓ answer                  (4)             </p> <p>                 ✓ AM = 106,01                  ✓ DM = 66,01                  ✓ cos rule                  ✓ answer                  (4)             </p> <p style="text-align: right;"><b>[10]</b></p>
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**QUESTION 12**

12.1



tan 2x:  
 ✓ asymptotes (45° & -45°)  
 ✓ x-intercepts  
 ✓ middle curve  
 ✓ end curves  
**Note:** The curve must tend toward the asymptotes

2 cos x:  
 ✓ y-intercept  
 ✓ x-intercepts

(6)

**Note:**  
 If the candidate draws outside of the interval, NO PENALTY.

<p>12.2 For these values of <math>x</math>, <math>\cos 2x \neq 0</math></p> $2 \cos x = \frac{\sin 2x}{\cos 2x}$ $= \frac{2 \sin x \cos x}{1 - 2 \sin^2 x}$ $1 - 2 \sin^2 x = \sin x \text{ or } \cos x = 0$ $2 \sin^2 x + \sin x - 1 = 0 \text{ or } \cos x = 0$ $(\sin x + 1)(2 \sin x - 1) = 0$ $\sin x = -1 \text{ or } \sin x = \frac{1}{2} \text{ or } \cos x = 0$ $x = \pm 90^\circ \text{ or } x = 30^\circ$ <p><b>OR</b></p> <p>For these values of <math>x</math>, <math>\cos 2x \neq 0</math></p> $2 \cos x = \frac{\sin 2x}{\cos 2x}$ $= \frac{2 \sin x \cos x}{2 \cos^2 x - 1}$ $2 \cos x(2 \cos^2 x - 1) = 2 \sin x \cos x$ $2 \cos x(2(1 - \sin^2 x) - 1) = 2 \sin x \cos x$ $2 \cos x(1 - 2 \sin^2 x) - 2 \sin x \cos x = 0$ $2 \cos x(2 \sin^2 x + \sin x - 1) = 0$ $2 \sin^2 x + \sin x - 1 = 0 \text{ or } \cos x = 0$ $(\sin x + 1)(2 \sin x - 1) = 0$ $\sin x = -1 \text{ or } \sin x = \frac{1}{2} \text{ or } \cos x = 0$ $x = \pm 90^\circ \text{ or } x = 30^\circ$ <p><b>OR</b></p> $2 \cos x = \frac{\sin 2x}{\cos 2x}$ $2 \cos x \cdot \cos 2x = \sin 2x$ $2 \cos x \cdot \cos 2x - 2 \sin x \cdot \cos x = 0$ $2 \cos x(\cos 2x - \sin x) = 0$ $\cos 2x = \sin x \text{ or } 2 \cos x = 0$ $1 - 2 \sin^2 x = \sin x$ $2 \sin^2 x + \sin x - 1 = 0$ $(\sin x + 1)(2 \sin x - 1) = 0$ $\sin x = -1 \text{ or } \sin x = \frac{1}{2} \text{ or } \cos x = 0$ $x = \pm 90^\circ \text{ or } x = 30^\circ$	$\checkmark \frac{\sin 2x}{\cos 2x}$ $\checkmark 2 \sin x \cos x$ $\checkmark 1 - 2 \sin^2 x$ $\checkmark \cos x = 0$ $\checkmark \text{factors}$ $\checkmark \text{equations}$ $\checkmark \pm 90^\circ$ $\checkmark 30^\circ$ <p style="text-align: right;">(8)</p> $\checkmark \frac{\sin 2x}{\cos 2x}$ $\checkmark 2 \sin x \cos x$ $\checkmark 2 \cos^2 x - 1$ $\checkmark \cos x = 0$ $\checkmark \text{factors}$ $\checkmark \text{equations}$ $\checkmark \pm 90^\circ$ $\checkmark 30^\circ$ <p style="text-align: right;">(8)</p> $\checkmark \frac{\sin 2x}{\cos 2x}$ $\checkmark 2 \sin x \cos x$ $\checkmark 1 - 2 \sin^2 x$ $\checkmark \cos x = 0$ $\checkmark \text{factors}$ $\checkmark \text{equations}$ $\checkmark \pm 90^\circ$ $\checkmark 30^\circ$ <p style="text-align: right;">(8)</p>
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	<p><b>OR</b></p> $2 \cos x = \frac{\sin 2x}{\cos 2x}$ $2 \cos x \cdot \cos 2x = \sin 2x$ $2 \cos x \cdot \cos 2x - 2 \sin x \cdot \cos x = 0$ $2 \cos x (\cos 2x - \sin x) = 0$ $\cos 2x = \sin x \quad \text{or} \quad 2 \cos x = 0$ $\cos 2x = \cos(90^\circ - x) \quad x = 90^\circ$ $2x = \pm(90^\circ - x) + k \cdot 360^\circ \quad k \in \mathbb{Z}$ $3x = 90^\circ + k \cdot 360^\circ$ $x = 30^\circ + k \cdot 120^\circ$ $x = -90^\circ \quad \text{or} \quad x = 30^\circ$ <p style="text-align: center;"><i>or</i></p> $x = -90^\circ + k \cdot 360^\circ$ $x = -90^\circ$	<ul style="list-style-type: none"> <li>✓ <math>\frac{\sin 2x}{\cos 2x}</math></li> <li>✓ <math>2 \sin x \cos x</math></li> <li>✓ <math>\sin x = \cos(90^\circ - x)</math></li>   <li>✓ <math>\cos x = 0</math></li>   <li>✓ factors</li> <li>✓ equations</li> <li>✓ <math>\pm 90^\circ</math></li> <li>✓ <math>30^\circ</math></li>   <li>(8)</li> </ul>
12.3	$0^\circ < x < 45^\circ$ Or $-90^\circ < x < -45^\circ$	<ul style="list-style-type: none"> <li>✓✓ critical points</li> <li>✓✓ notation</li> </ul> <p style="text-align: right;">(4)</p>
12.4	Period = $2(360^\circ) = 720^\circ$	<ul style="list-style-type: none"> <li>✓✓ answer</li> </ul> <p style="text-align: right;">(2)</p>
12.5	$x = -45^\circ + 25^\circ = -20^\circ$ $x = 45^\circ + 25^\circ = 70^\circ$  <b>OR</b> $2(x - 25^\circ) = -90^\circ$ $2(x - 25^\circ) = 90^\circ$ $2x - 50^\circ = -90^\circ$ and $2x - 50^\circ = 90^\circ$ $2x = -40^\circ$ $2x = 140^\circ$ $x = -20^\circ$ $x = 70^\circ$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> <p><b>Note:</b> Answer only: <b>full marks</b></p> </div> <ul style="list-style-type: none"> <li>✓ <math>x = -20^\circ</math></li> <li>✓ <math>x = 70^\circ</math></li> </ul> <p style="text-align: right;">(2)</p> <p style="text-align: right;"><b>[22]</b></p>

**TOTAL : 150**