basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2010

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, 4 diagram sheets and 1 information sheet.

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This paper consists of 12 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.

4. Answers only will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. Round off to TWO decimal places if necessary, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. FOUR diagram sheets for QUESTION 1.2, QUESTION 2.1, QUESTION 2.2, QUESTION 7.1 and QUESTION 12.1 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.

9. An information sheet, with formulae, is included at the end of this question paper.

10. Number the answers correctly according to the numbering system used in this question paper.

11. Write legibly and present your work neatly.

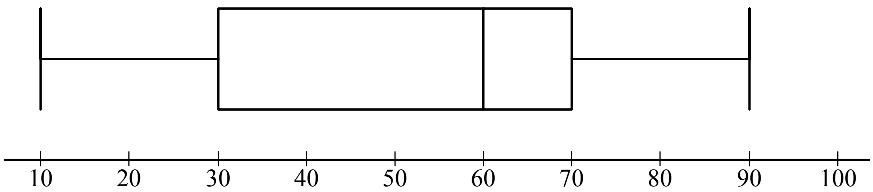
QUESTION 1

Two Mathematics classes, A and B, are in competition to see which class performed best in the June examination. The marks of the learners in Class A are given below and the box and whisker diagram below illustrates the results of Class B. Both classes have 25 learners. (Marks are given in %.)

The marks of the learners in Class A are:

9	14	14	19	21
23	33	35	37	37
42	45	55	56	57
59	68	75	75	75
77	78	80	81	92

The box and whisker diagram for the learners in Class B is:



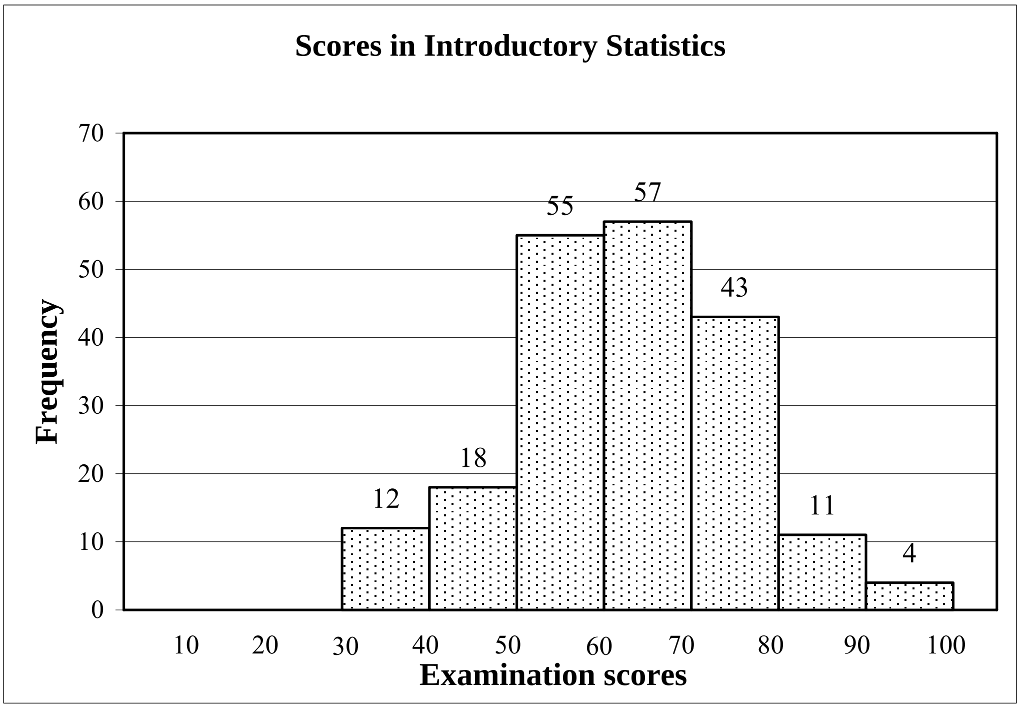
1.1 Write down the five-number summary for Class A. (4)

1.2 Draw the box and whisker diagram that represents Class A's marks on DIAGRAM SHEET 1. Clearly indicate ALL relevant values. (2)

1.3 Determine which class performed better in the June examination and give reasons for your conclusion. (3)
[9]

QUESTION 2

The histogram below shows the distribution of examination scores for 200 learners in Introductory Statistics.



2.1 Complete the cumulative frequency table for the above data provided on DIAGRAM SHEET 2. (2)

2.2 Draw an ogive of the above data on the grid provided on DIAGRAM SHEET 2. (5)

2.3 Use the ogive to estimate how many learners scored 75% or more for the examination. (1)
[8]

QUESTION 3

The owner of an ice-cream parlour gathered information on the average sales per day of litres of ice-cream during a festival. The table below shows a summary for 12 days.

Day	1	2	3	4	5	6	7	8	9	10	11	12
Averages sales of ice-cream (litres)	217	211	221	239	144	161	168	185	265	249	160	184

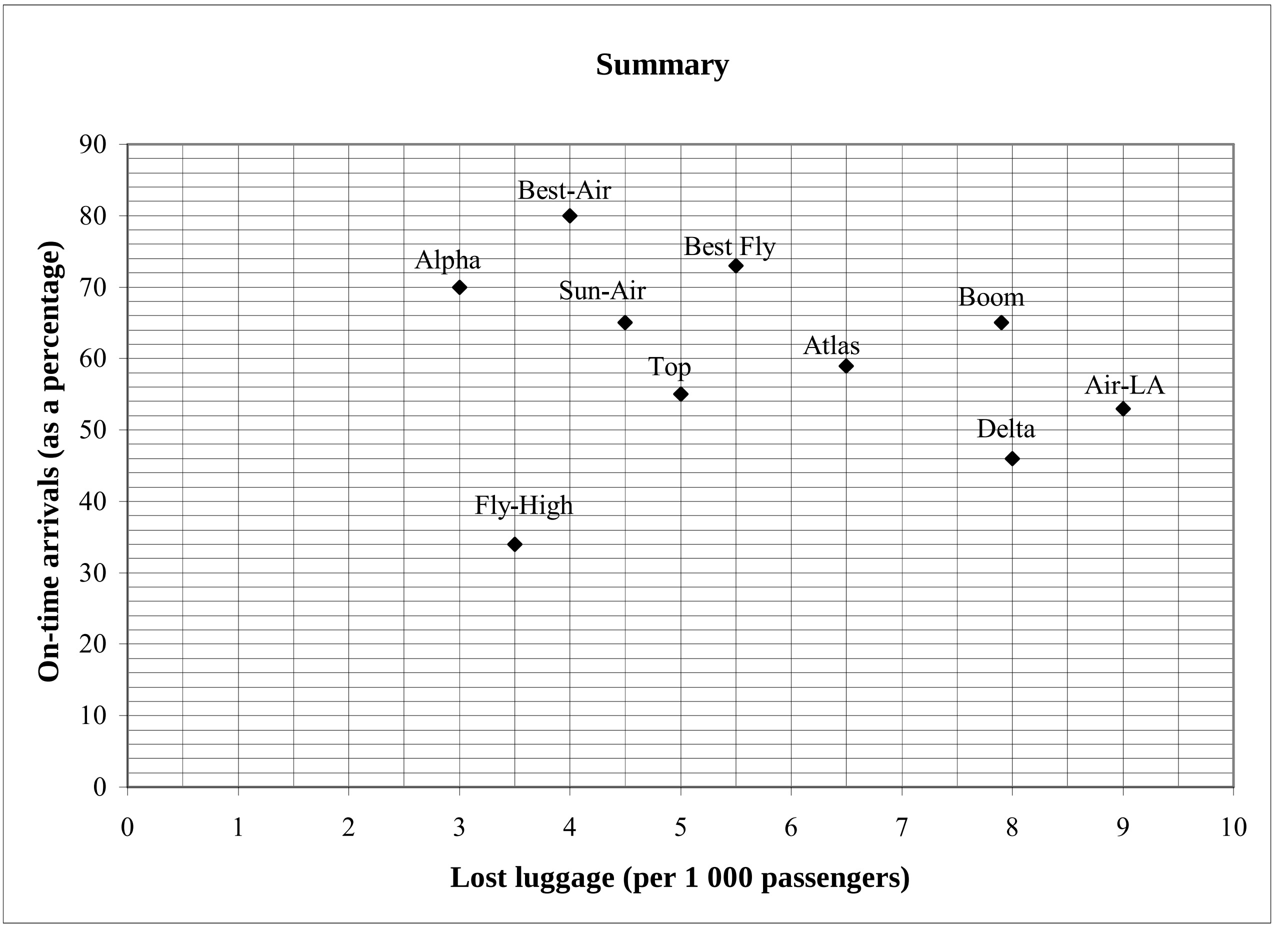
3.1 Calculate the mean number of litres of ice-cream that the parlour sells per day during the festival. (2)

3.2 Calculate the standard deviation of the given information. (3)

3.3 What is the maximum number of litres of ice-cream that the owner must stock per day in order to be within ONE standard deviation of the mean? (2)
[7]

QUESTION 4

A researcher suspects that airlines, whose planes arrive on time, are less likely to lose the luggage of their passengers. Information gathered from 10 airline companies is summarised in the grid below.



Use the scatter plot to answer the following questions.

4.1 Which airline has the worst record for on-time arrivals? (1)

4.2 Is the following statement likely to be TRUE? Motivate your answer.

Of 5 120 passengers transported by Boom airlines, 40 passengers lost their luggage. (1)

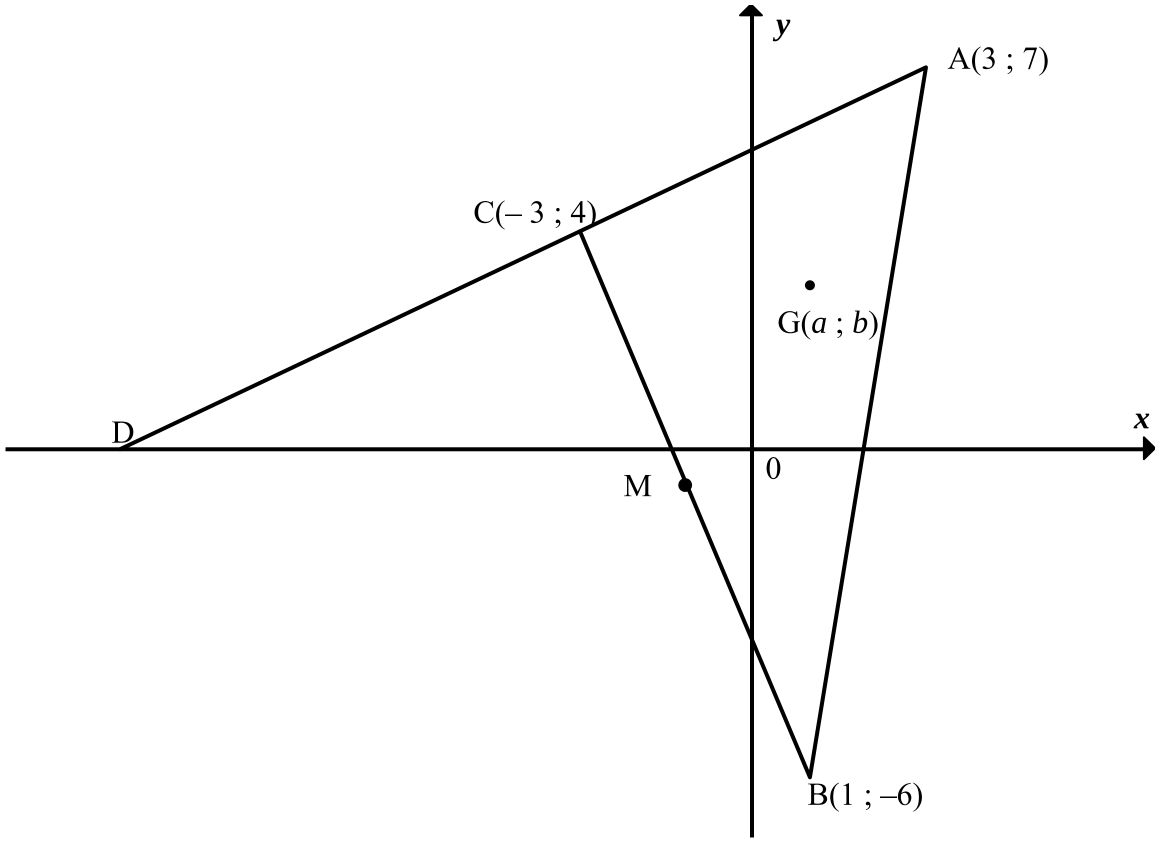
4.3 Does the data confirm the researcher's suspicions? Justify your answer. (2)

4.4 Which ONE of the 10 airlines would you prefer to use? Give a reason for your answer. (2)

[6]

QUESTION 5

In the diagram below, A, B and C are the vertices of a triangle. AC is extended to cut the x-axis at D.



5.1 Calculate the gradient of:

5.1.1 AD (2)

5.1.2 BC (1)

5.2 Calculate the size of $D\hat{C}B$. (3)

5.3 Write down an equation of the straight line AD. (2)

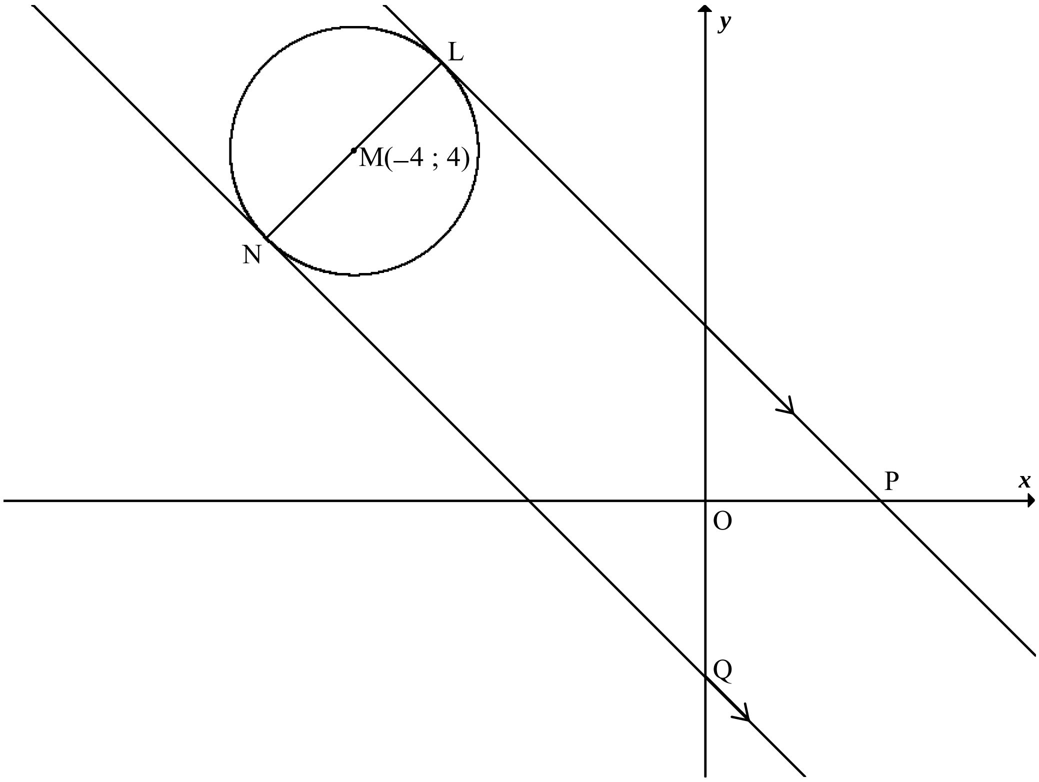
5.4 Determine the coordinates of M, the midpoint of BC. (2)

5.5 If G(a ; b) is a point such that A, G and M lie on the same straight line, show that $b = 2a + 1$. (4)

5.6 Hence calculate TWO possible values of b if $GC = \sqrt{17}$. (6)

[20]

QUESTION 6



The line LP, with equation $y + x - 2 = 0$, is a tangent at L to the circle with centre M(– 4 ;4). LN is a diameter of the circle. Also LP || NQ, where P lies on the x-axis, and Q lies on the y-axis.

6.1 Determine the equation of the diameter LN. (3)

6.2 Calculate the coordinates of L. (2)

6.3 Determine the equation of the circle. (3)

6.4 Write down the coordinates of N. (3)

6.5 Write down the equation of NQ. (3)

6.6 If the length of the diameter is doubled and the circle is translated horizontally 6 units to the right, write down the equation of the new circle. (3)

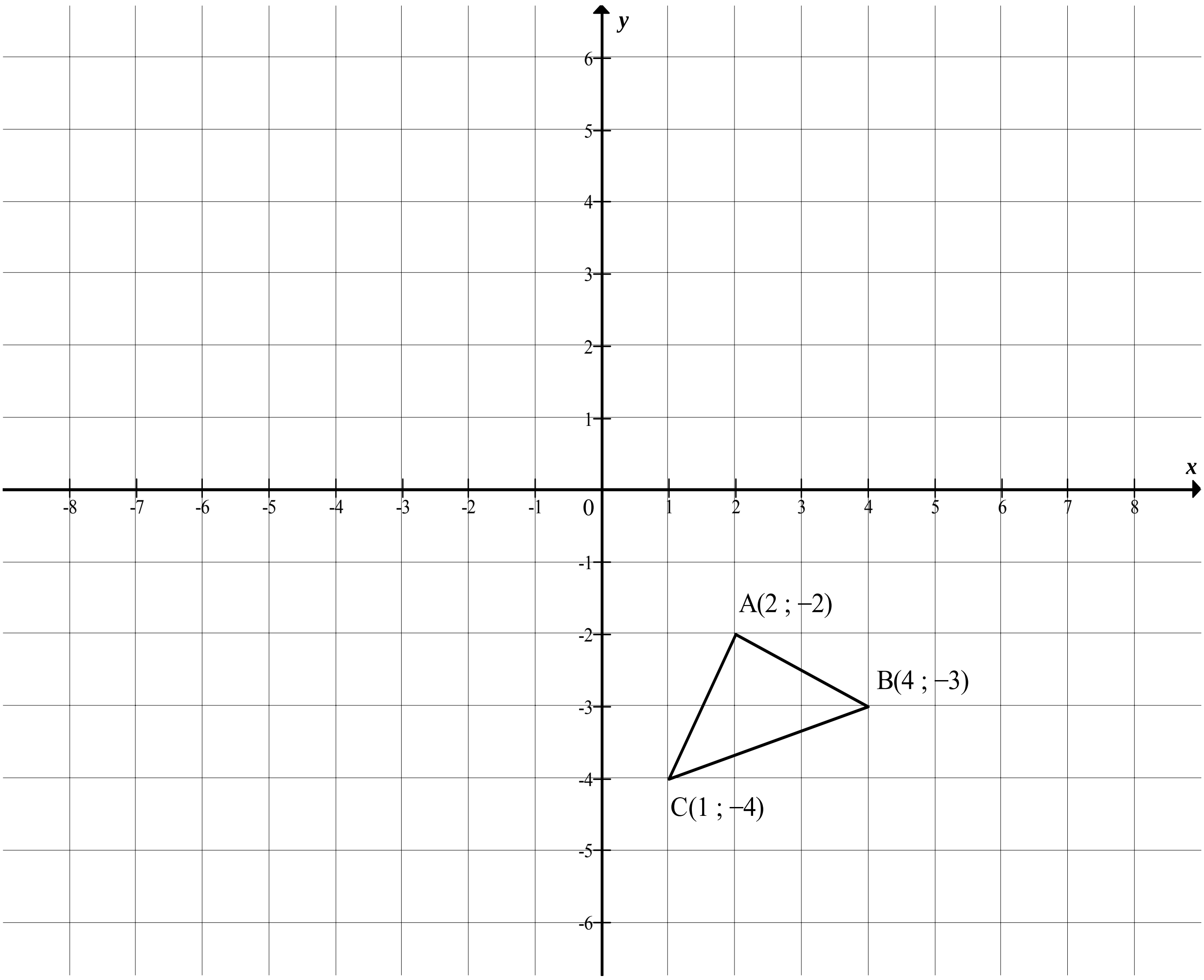
[17]

QUESTION 7

A transformation T is described as follows:

- A reflection in the *x*-axis, followed by
- A translation of 4 units left and 2 units down, followed by
- An enlargement through the origin by a factor of 2

In the diagram ΔABC is given with vertices A(2 ; –2), B(4 ; –3) and C(1 ; –4).



7.1 If ΔABC is transformed by T to $\Delta A^{/}B^{/}C^{/}$ (in that order), use the grid provided on DIAGRAM SHEET 3 to sketch $\Delta A^{/}B^{/}C^{/}$. Show ALL the steps. (6)

7.2 Write down the general rule for (*x*; *y*) under transformation T in the form $(x\,;y) \rightarrow \ldots$ (4)

7.3 Calculate the area of $\Delta A^{/}B^{/}C^{/}$. (4)

[14]

QUESTION 8

8.1 The point K(2 ; 4) is rotated about the origin through an angle of 75°, in an anticlockwise direction. Without the use of a calculator, determine the x-coordinate of the image $K^{/}$ of K. Simplify your answer. (6)

8.2 The point (3 ; 1) is rotated in an anticlockwise direction about the origin through an angle β. If the image is $\left(\frac{3-\sqrt{3}}{2}\ ;\ \frac{1+3\sqrt{3}}{2}\right)$, calculate β. (6)

[12]

QUESTION 9

Given: $\tan\alpha = \frac{3}{4}$; where $\alpha \in [0°\ ;\ 90°]$

With the use of a sketch and without the use of a calculator, calculate:

9.1 $\sin\alpha$ (3)

9.2 $\cos^2(90° - \alpha) - 1$ (2)

9.3 $1 - \sin 2\alpha$ (3)

[8]

QUESTION 10

(You may NOT use a calculator to answer this question.)

10.1 Simplify completely:

$$\frac{\sin(90° + \theta) + \cos(180° + \theta)\sin(-\theta)}{\sin 180° - \tan 135°}$$ (5)

10.2 Prove that for any angle A:

$$\frac{4\sin A\cos A\cos 2A\sin 15°}{\sin 2A\left(\tan 225° - 2\sin^2 A\right)} = \frac{\sqrt{6}-\sqrt{2}}{2}$$ (6)

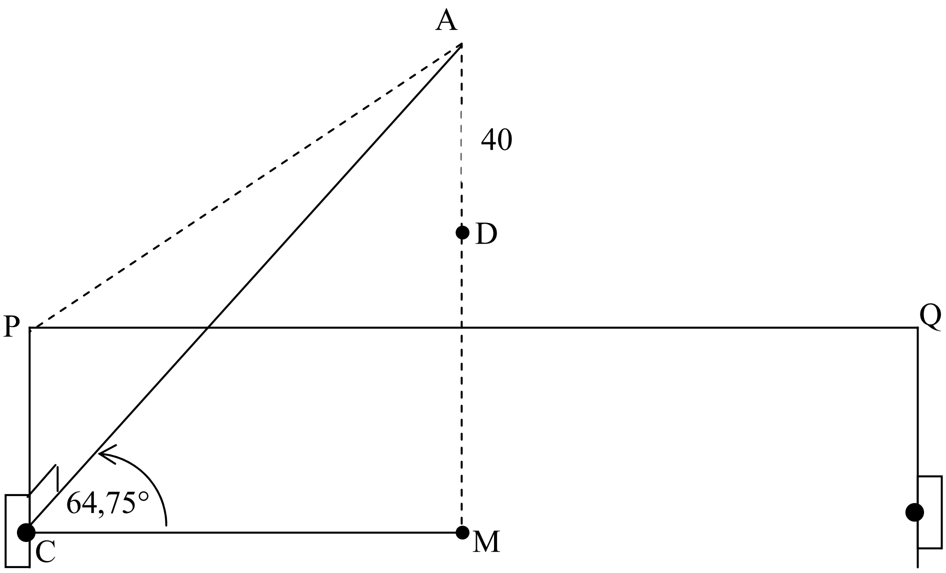
10.3 Determine the general solution of:

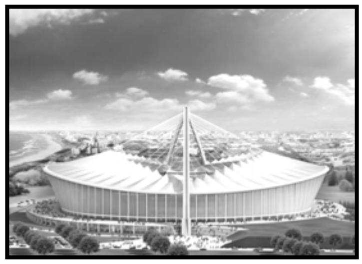
$$6\cos x - 5 = \frac{4}{\cos x} \quad ; \quad \cos x \neq 0$$ (6)

[17]

QUESTION 11

The angle of elevation from a point C on the ground, at the centre of the goalpost, to the highest point A of the arc, directly above the centre of the Moses Mabhida soccer stadium, is 64,75°. The soccer pitch is 100 metres long and 64 metres wide as prescribed by FIFA for world cup stadiums. Also AC ⊥ PC. In the figure below PQ = 100 metres and PC = 32 metres.





11.1 Determine AC. (3)

11.2 Calculate $P\hat{A}C$. (3)

11.3 A camera is positioned at point D, 40 metres directly below A. Calculate the distance from D to C. (4)
[10]

QUESTION 12

Given: $f(x) = 2\cos x$ and $g(x) = \tan 2x$

12.1 Sketch the graphs of f and g on the same system of axes provided on DIAGRAM SHEET 4, for $x \in [-90°\ ;\ 90°]$ (6)

12.2 Solve for x if $2\cos x = \tan 2x$ and $x \in [-90°\ ;\ 90°]$. Show ALL working details. (8)

12.3 Use the graph to solve for x: $2\cos x . \tan 2x > 0$. (4)

12.4 Write down the period of $f\left(\frac{x}{2}\right)$. (2)

12.5 Write down the equations of the asymptotes of $g(x - 25°)$, where $x \in [-90°\ ;\ 90°]$. (2)
[22]

TOTAL: 150

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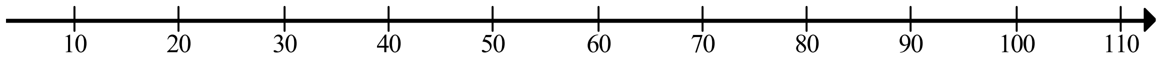
CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 1.2

Class A



Class B

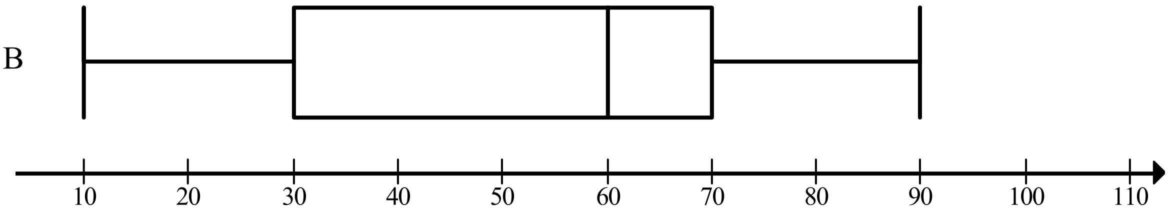


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DIAGRAM SHEET 2

QUESTION 2.1

EXAMINATION SCORE (x)	FREQUENCY	CUMULATIVE FREQUENCY
$30 \leq x < 40$	12	
$40 \leq x < 50$	18	
$50 \leq x < 60$	55	
$60 \leq x < 70$	57	
$70 \leq x < 80$	43	
$80 \leq x < 90$	11	
$90 \leq x < 100$	4	

QUESTION 2.2

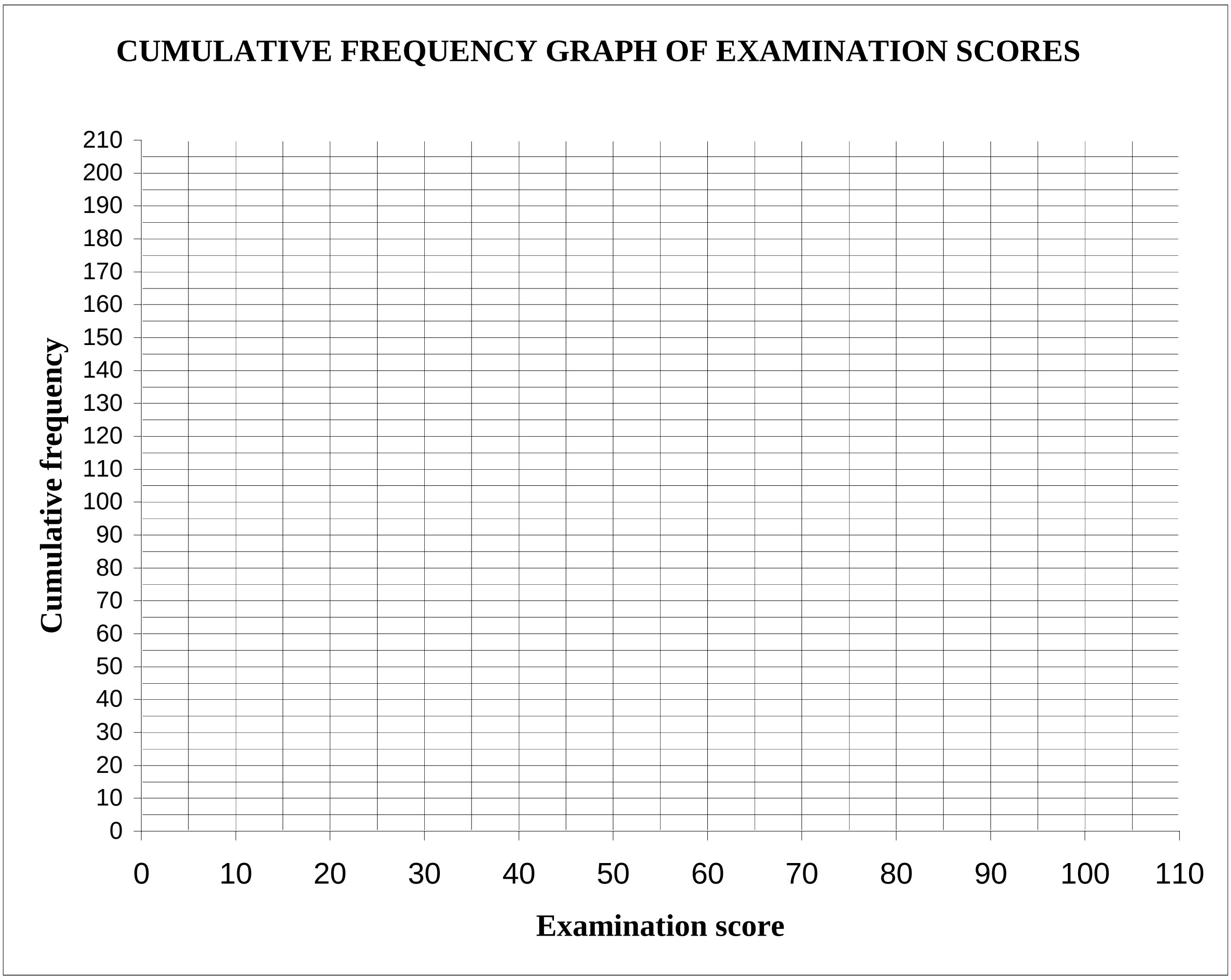

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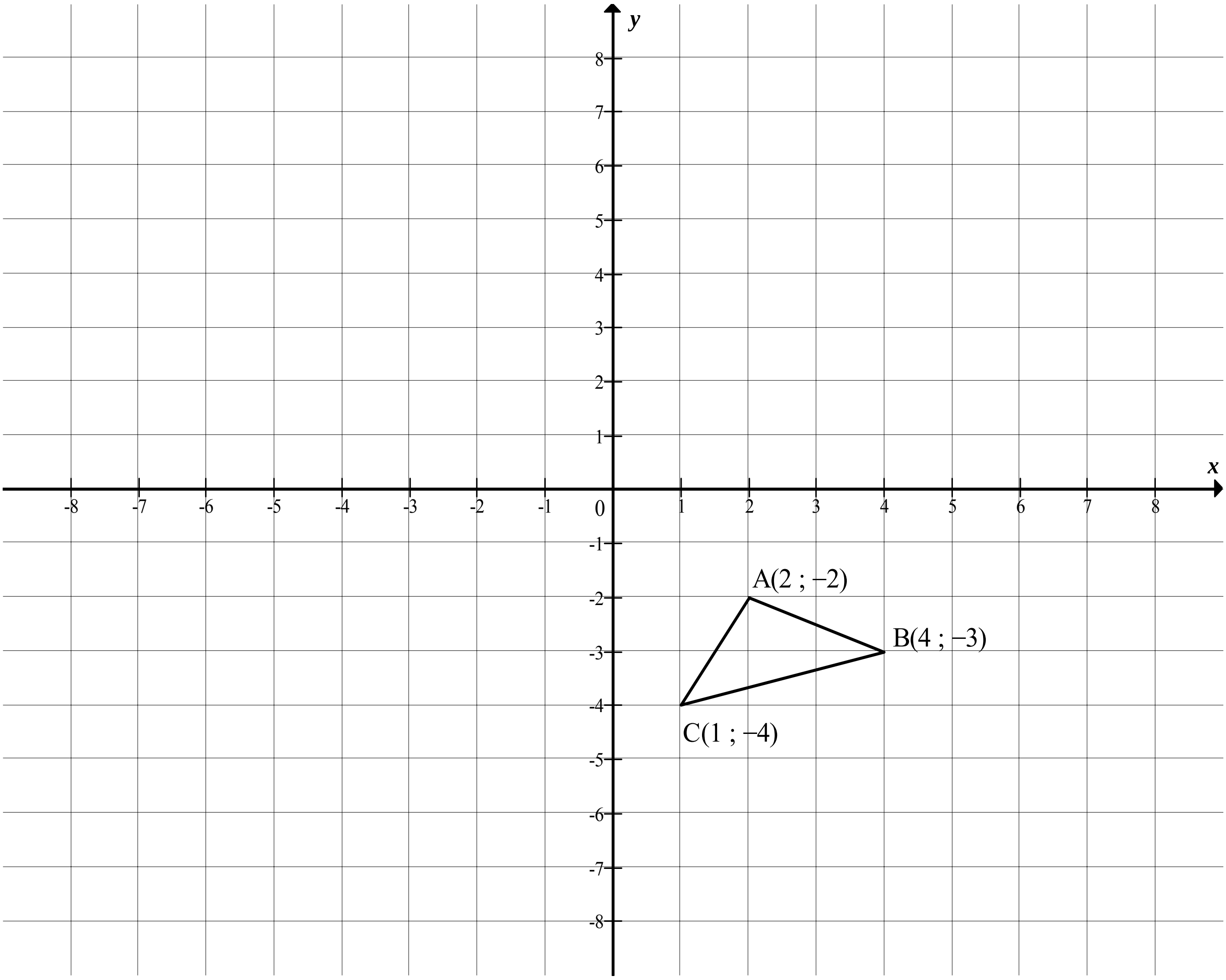
DIAGRAM SHEET 3

QUESTION 7.1



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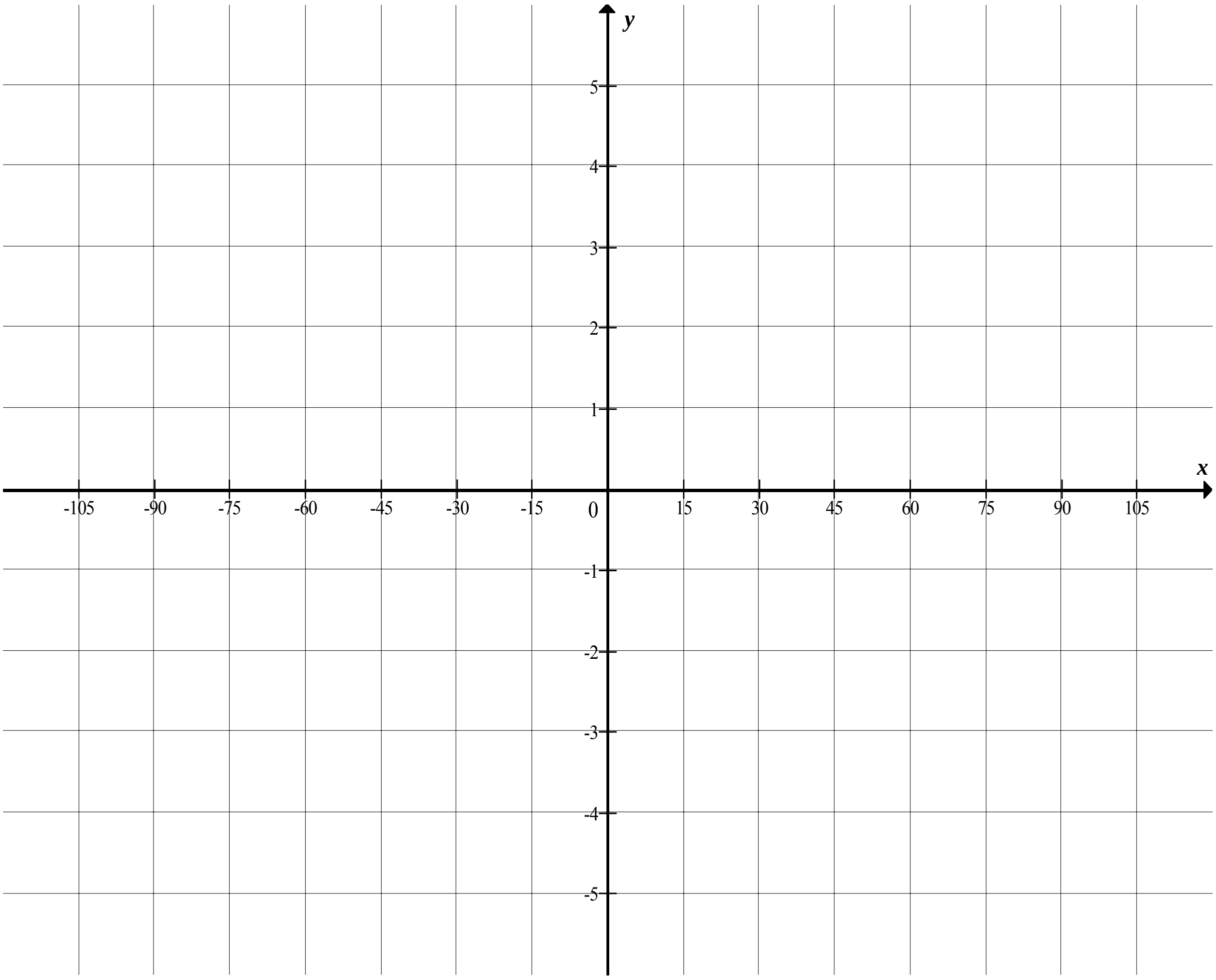
EXAMINATION NUMBER:

DIAGRAM SHEET 4

QUESTION 12.1



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INFORMATION SHEET: MATHEMATICS
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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$\text{area } \Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x ; y) \to (x\cos\theta + y\sin\theta ; y\cos\theta - x\sin\theta)$ $\quad (x ; y) \to (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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