basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2010

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages, 2 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round answers off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. TWO diagram sheets for answering QUESTION 5.3, QUESTION 6.4 and QUESTION 11.2 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.

9. An information sheet, with formulae, is included at the end of the question paper.

10. Number the answers correctly according to the numbering system used in this question paper.

11. Write legibly and present your work neatly.

QUESTION 1

1.1 Solve for x, correct to TWO decimal places, where necessary:

1.1.1 $(3-x)(5-x)=3$ (3)

1.1.2 $3x^2 = 2(x+2)$ (4)

1.1.3 $4+5x > 6x^2$ (4)

1.2 Solve for x and y simultaneously:

$3y = 2x$
$x^2 - y^2 + 2x - y = 1$ (7)

1.3 Calculate the integer that is the closest approximation to:

$\dfrac{5^{2007}+5^{2010}}{5^{2008}+5^{2009}}$ (Show ALL workings.) (3)

[21]

QUESTION 2

2.1 Evaluate: $\sum_{n=1}^{20} 3^{n-2}$ (4)

2.2 The following sequence forms a convergent geometric sequence: $5x\ ;\ x^2\ ;\ \dfrac{x^3}{5}\ ;\ \ldots$

2.2.1 Determine the possible values of x. (3)

2.2.2 If $x = 2$, calculate S_∞. (2)

2.3 The following arithmetic sequence is given: 20 ; 23 ; 26 ; 29; … ; 101

2.3.1 How many terms are there in this sequence? (2)

2.3.2 The even numbers are removed from the sequence.
Calculate the sum of the terms of the remaining sequence. (6)

[17]

QUESTION 3

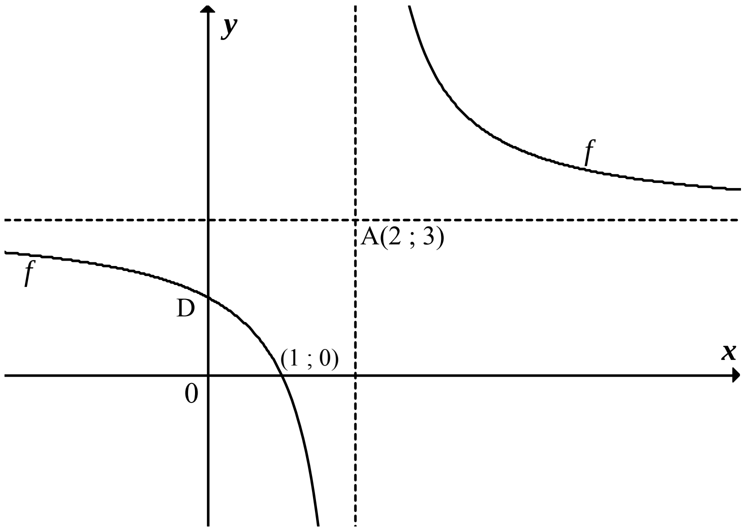
The sequence $4\ ;\ 9\ ;\ x\ ;\ 37; \ldots$ is a quadratic sequence.

3.1 Calculate x. (3)

3.2 Hence, or otherwise, determine the n^{th} term of the sequence. (4)
[7]

QUESTION 4

Given $f(x) = \dfrac{a}{x-p} + q$. The point A(2 ; 3) is the point of intersection of the asymptotes of f.
The graph of f intersects the x-axis at (1 ; 0).
D is the y-intercept of f.



4.1 Write down the equations of the asymptotes of f. (2)

4.2 Determine an equation of f. (3)

4.3 Write down the coordinates of D. (2)

4.4 Write down an equation of g if g is the straight line joining A and D. (3)

4.5 Write down the coordinates of the other point of intersection of f and g. (4)
[14]

QUESTION 5

Consider the function $f(x) = 4^{-x} - 2$.

5.1 Calculate the coordinates of the intercepts of f with the axes. (4)

5.2 Write down the equation of the asymptote of f. (1)

5.3 Sketch the graph of f on DIAGRAM SHEET 1. (3)

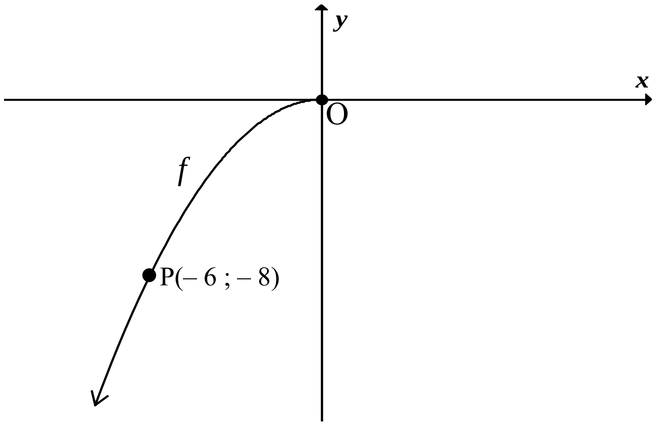
5.4 Write down the equation of g if g is the graph of f shifted 2 units upwards. (1)

5.5 Solve for x if $f(x) = 3$. (You need not simplify your answer.) (3)

[12]

QUESTION 6

The graph of $f(x) = ax^2$, $x \leq 0$ is sketched below.
The point P(– 6 ; – 8) lies on the graph of f.



6.1 Calculate the value of a. (2)

6.2 Determine the equation of f^{-1}, in the form $y = \ldots$ (3)

6.3 Write down the range of f^{-1}. (1)

6.4 Draw the graph of f^{-1} on DIAGRAM SHEET 1. Indicate the coordinates of a point on the graph different from (0 ; 0). (2)

6.5 The graph of f is reflected across the line $y = x$ and thereafter it is reflected across the x-axis. Determine the equation of the new function in the form $y = \ldots$ (3)

[11]

QUESTION 7

7.1 At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years? (5)

7.2 Timothy buys furniture to the value of R10 000. He borrows the money on 1 February 2010 from a financial institution that charges interest at a rate of 9,5% p.a. compounded monthly. Timothy agrees to pay monthly instalments of R450. The agreement of the loan allows Timothy to start paying these equal monthly instalments from 1 August 2010.

7.2.1 Calculate the total amount owing to the financial institution on 1 July 2010. (2)

7.2.2 How many months will it take Timothy to pay back the loan? (4)

7.2.3 What is the balance of the loan immediately after Timothy has made the 25th payment? (3)

[14]

QUESTION 8

8.1 Differentiate $g(x) = x^2 - 5$ from first principles. (5)

8.2 Evaluate $\dfrac{dy}{dx}$ if $y = \dfrac{x^6}{2} + 4\sqrt{x}$. (3)

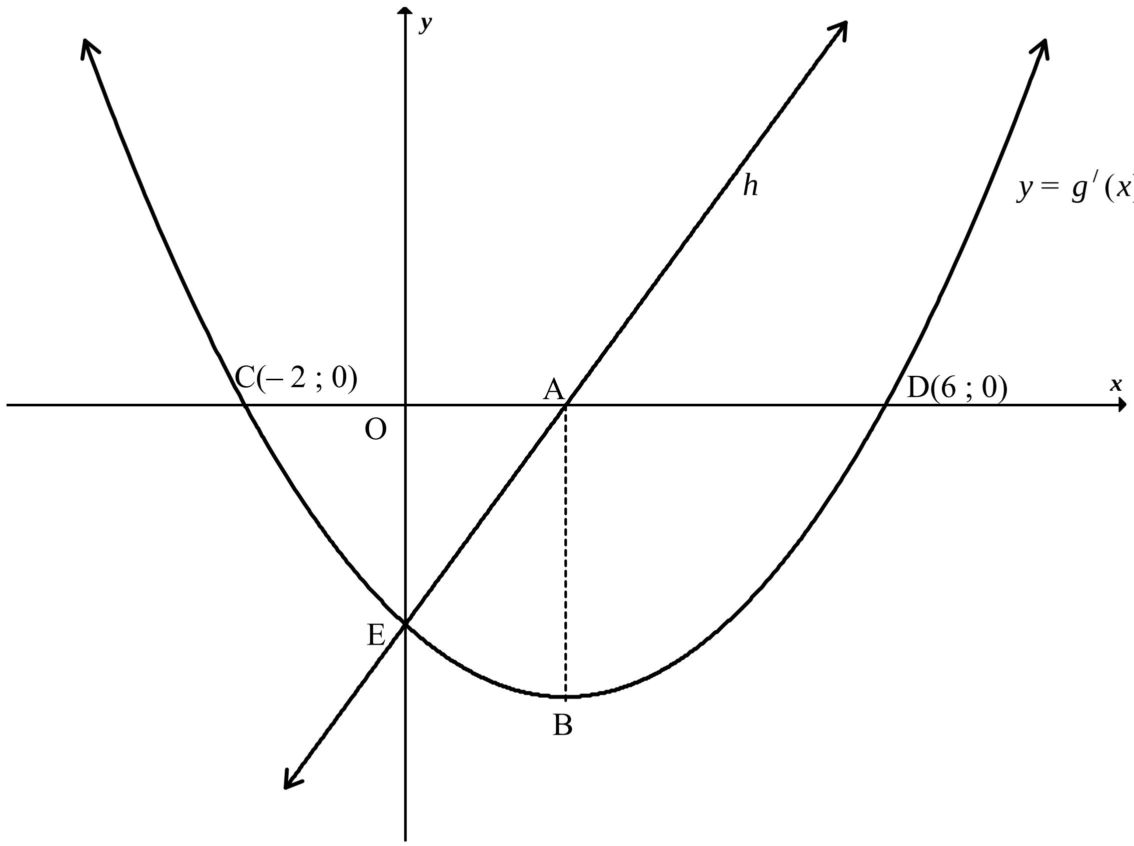
8.3 A function $g(x) = ax^2 + \dfrac{b}{x}$ has a minimum value at $x = 4$. The function value at $x = 4$ is 96. Calculate the values of a and b. (6)

[14]

QUESTION 9

The graphs of $y = g'(x) = ax^2 + bx + c$ and $h(x) = 2x - 4$ are sketched below. The graph of $y = g'(x) = ax^2 + bx + c$ is the derivative graph of a cubic function g.

The graphs of h and g' have a common y-intercept at E.
C(– 2 ; 0) and D(6 ; 0) are the x-intercepts of the graph of g'.
A is the x-intercept of h and B is the turning point of g'.
AB || y-axis.



9.1 Write down the coordinates of E. (1)

9.2 Determine the equation of the graph of g' in the form $y = ax^2 + bx + c$. (4)

9.3 Write down the x-coordinates of the turning points of g. (2)

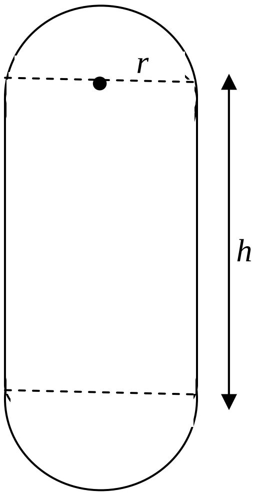
9.4 Write down the x-coordinate of the point of inflection of the graph of g. (2)

9.5 Explain why g has a local maximum at $x = -2$. (3)
[12]

QUESTION 10

A satellite is to be constructed in the shape of a cylinder with a hemisphere at each end. The radius of the cylinder is r metres and its height is h metres (see diagram below). The outer surface area of the satellite is to be coated with heat-resistant material which is very expensive.

The volume of the satellite has to be $\frac{\pi}{6}$ cubic metres.



Outer surface area of a sphere = $4\pi r^2$
Curved surface area of a cylinder = $2\pi rh$
Volume of a sphere = $\frac{4}{3}\pi r^3$
Volume of a cylinder = $\pi r^2 h$

10.1 Show that $h = \frac{1}{6r^2} - \frac{4r}{3}$. (3)

10.2 Hence, show that the outer surface area of the satellite can be given as $S = \frac{4\pi r^2}{3} + \frac{\pi}{3r}$. (3)

10.3 Calculate the minimum outer surface area of the satellite. (6)
[12]

QUESTION 11

A factory produces two types of braai stands, Type **A** and Type **B**.

- Type A requires one hour of machine-time and three hours for welding and finishing.
- Type B requires two hours of machine-time and one hour for welding and finishing.
- In one day the factory has available no more than 28 hours machine-time and no more than 24 hours for welding and finishing.

11.1 If the factory produces x Type A and y Type B braai stands on a particular day, write down the relevant constraints in terms of x and y. (4)

11.2 Represent the system of constraints on the graph paper provided on DIAGRAM SHEET 2. Indicate the feasible region by means of shading. (3)

11.3 Now determine the largest number of the following types that could be manufactured in one day:

11.3.1 Type A (1)

11.3.2 Type B (1)

11.4 Determine how many Type A and Type B braai stands should be manufactured each day for the factory to produce the maximum number of braai stands. (2)

11.5 If the demand for Type A braai stands is at least as large as the demand for Type B braai stands, calculate the largest number of braai stands that can be manufactured in one day and the machine-time required in this case. (5)

[16]

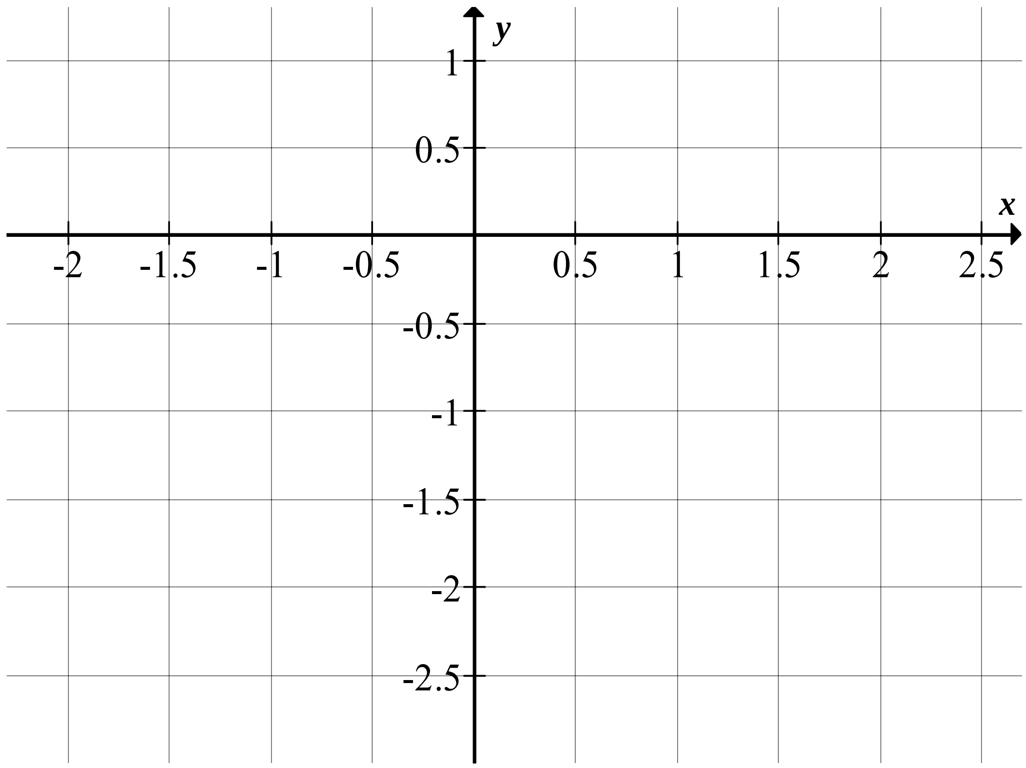
TOTAL: 150

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 5.3



QUESTION 6.4

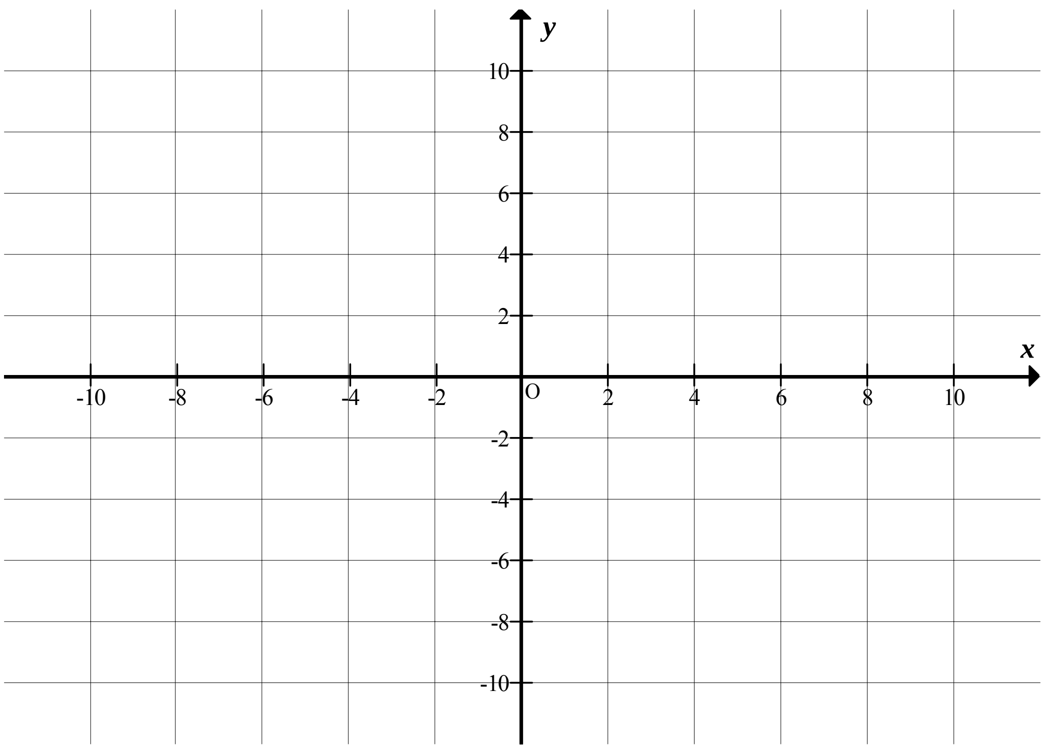


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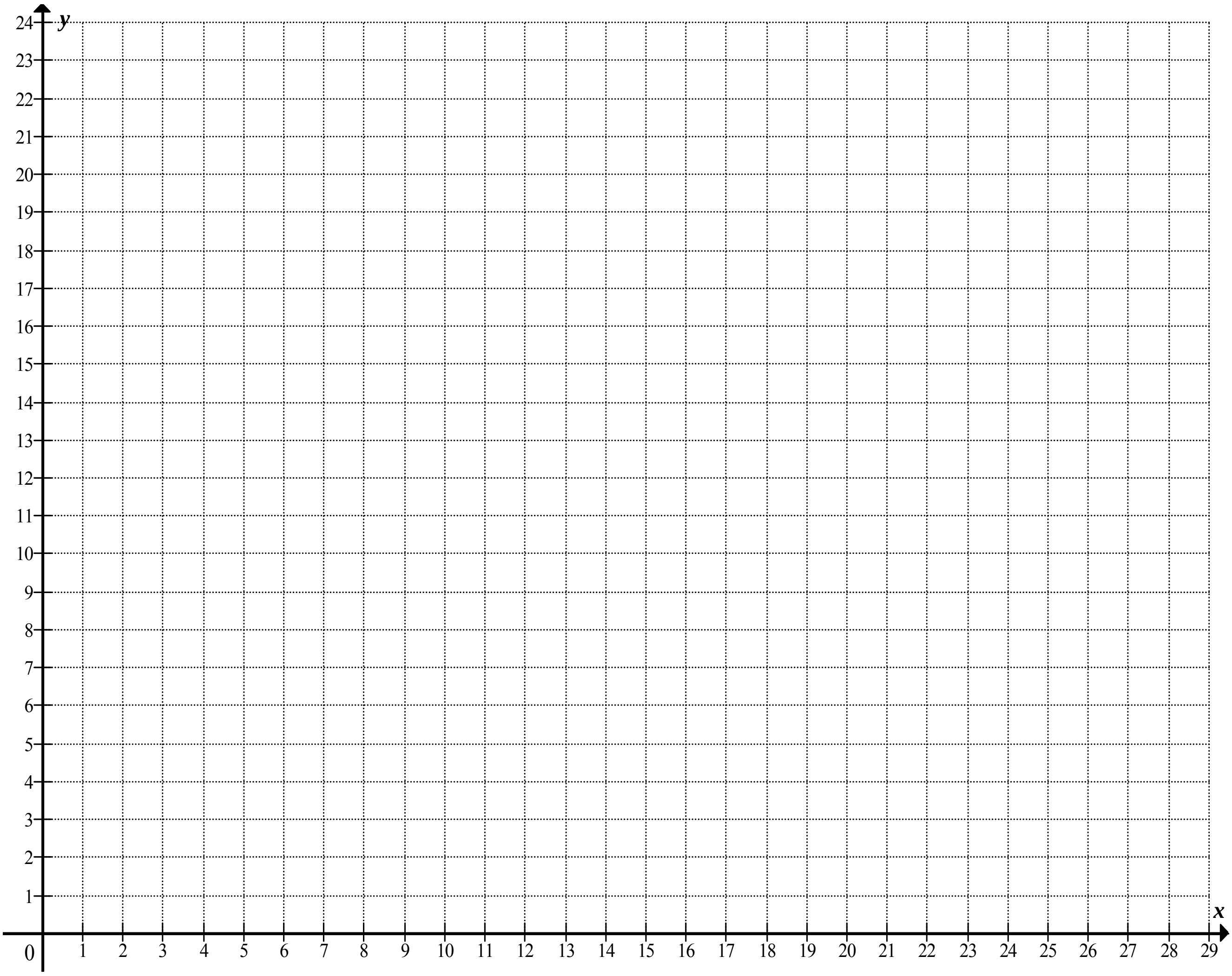
DIAGRAM SHEET 2

QUESTION 11.2



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x ; y) \to (x\cos\theta + y\sin\theta ; y\cos\theta - x\sin\theta)$ $\quad (x ; y) \to (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$